

Math 417 – Section 35 Solutions

1. (b) Find all values of $\tan^{-1}(1+i)$.

$$\begin{aligned}\tan^{-1}(1+i) &= \frac{i}{2} \log \left(\frac{i+(1+i)}{i-(1+i)} \right) \\ &= \frac{i}{2} \log(-1-2i) \\ &= \frac{i}{2} \left[\ln \sqrt{5} + i(\tan^{-1} 2 - \pi + 2n\pi) \right], \quad n = 0, \pm 1, \pm 2, \dots \\ &= -\frac{1}{2}(\tan^{-1} 2 - \pi + 2n\pi) + \frac{i}{4}(\ln 5), \quad n = 0, \pm 1, \pm 2, \dots\end{aligned}$$

where we have used the fact that the principal argument of $z = -1 - 2i$ is $\Theta = \tan^{-1} 2 - \pi$.