

Math 417 – Section 41 Solutions

1. The length of the arc C , which is the portion of the circle of radius 2 centered at $z = 0$ in the first quadrant, is $L = \pi$.

To put an upper bound on the integral:

$$\int_C \frac{dz}{z^2 - 1}$$

we must find an M such that $\left| \frac{1}{z^2 - 1} \right| \leq M$ on C . Using the fact that $|x - y| \geq ||x| - |y||$ we have:

$$|z^2 - 1| \geq ||z|^2 - |1|| = ||z|^2 - 1| = |2^2 - 1| = 3$$

Therefore,

$$\left| \frac{1}{z^2 - 1} \right| \leq \frac{1}{3}$$

and an upper bound on the integral is:

$$\left| \int_C \frac{dz}{z^2 - 1} \right| \leq ML = \frac{\pi}{3}$$

2. The length of C , the line segment from $z = i$ to $z = 1$, is $L = \sqrt{2}$.

To put an upper bound on the integral:

$$\int_C \frac{dz}{z^4}$$

we must find an M such that $\left| \frac{1}{z^4} \right| \leq M$ on C . We note that:

$$\left| \frac{1}{z^4} \right| = \frac{1}{|z|^4} \leq \frac{1}{(\sqrt{2}/2)^4} = 4$$

using the fact that the distance from the origin to the midpoint of C is $\sqrt{2}/2$. Therefore, an upper bound on the integral is:

$$\left| \int_C \frac{dz}{z^4} \right| \leq ML = 4\sqrt{2}$$

3. The length of C is the perimeter of the triangle which is $L = 3 + 4 + 5 = 12$.

An upper bound on $|e^z - \bar{z}|$ is:

$$|e^z - \bar{z}| \leq |e^z| + |\bar{z}| \leq e^x + |z| \leq e^0 + 4 = 5$$

Therefore, an upper bound on the integral is:

$$\left| \int_C (e^z - \bar{z}) dz \right| \leq ML = 60$$