

Math 417 – Sections 44–46 Solutions

1. (a) The function $f(z) = \frac{z^2}{z-3}$ is analytic everywhere except $z = 3$. However, since $z = 3$ lies outside of C , the Cauchy-Goursat Theorem applies and the integral is 0.
- (c) The function $f(z) = \frac{1}{z^2+2z+2}$ is analytic everywhere except $z = -1 \pm i$. However, since these points lie outside of C , the Cauchy-Goursat Theorem applies and the integral is 0.
- (e) The function $f(z) = \tan z = \frac{\sin z}{\cos z}$ is analytic everywhere except at $z = \frac{\pi}{2} + n\pi$ where $n = 0, \pm 1, \pm 2, \dots$. However, since these points lie outside of C , the Cauchy-Goursat Theorem applies and the integral is 0.
2. (a) The function $f(z) = \frac{1}{3z^2+1}$ is analytic everywhere except $z = \pm \frac{i}{\sqrt{3}}$. These points lie inside the square C_2 . Therefore, since $f(z)$ is analytic on C_1 and C_2 and at all points between the contours, the integrals over C_1 and C_2 are the same.
- (b) The function $f(z) = \frac{z+2}{\sin(z/2)}$ is analytic everywhere except $z = 2n\pi$ where $n = 0, \pm 1, \pm 2, \dots$. The point $z = 0$ lies inside C_2 and the remaining points lie outside C_1 . Therefore, since $f(z)$ is analytic on C_1 and C_2 and at all points between the contours, the integrals over C_1 and C_2 are the same.
- (c) The function $f(z) = \frac{z}{1-e^z}$ is analytic everywhere except $z = 0$. This point lies inside the square C_2 . Therefore, since $f(z)$ is analytic on C_1 and C_2 and at all points between the contours, the integrals over C_1 and C_2 are the same.
3. Let C be the rectangle $0 \leq x \leq 3, 0 \leq y \leq 2$. The function $f(z) = (z - (2+i))^{n-1}$ is singular only at $z = 2+i$ when $n = 0, -1, -2, \dots$. Therefore, we can deform the contour into a circle $|z - (2+i)| = R$ where $R \leq 1$. The integral of $f(z)$ over this new contour then follows.