

Math 417 – Sections 49 & 50 Solutions

6. The function $f(z) = (z + 1)^2$ is analytic everywhere on the boundary and in the interior of the region R . Furthermore, $f(z) \neq 0$ in R . Therefore, $|f(z)|$ attains a maximum and minimum value on the boundary of R .

Note that $|f(z)| = |(z + 1)^2| = |z + 1|^2$ represents the square of the distance between the point z and the point -1 . The point on R that is farthest from -1 is $z = 2$. Therefore,

$$\max |f(z)| = |2 + 1|^2 = 9$$

The point on R that is closest to -1 is $z = 0$. Therefore,

$$\min |f(z)| = |0 + 1|^2 = 1$$