

Math 417 – Sections 51 & 52 Solutions

1. To show that the sequence

$$z_n = -2 + i \frac{(-1)^n}{n^2} \quad n = 1, 2, \dots$$

converges to -2 , we first use the definition of the limit of a sequence.

Given $\varepsilon > 0$, we must find an n_0 such that:

$$|-2 - z_n| < \varepsilon \quad \text{whenever} \quad n > n_0$$

Note that:

$$\begin{aligned} |-2 - z_n| &< \varepsilon \\ \left| -2 - \left(-2 + i \frac{(-1)^n}{n^2} \right) \right| &< \varepsilon \\ \left| -i \frac{(-1)^n}{n^2} \right| &< \varepsilon \\ \frac{1}{n^2} &< \varepsilon \\ n &> \frac{1}{\sqrt{\varepsilon}} \end{aligned}$$

If we round the number $1/\sqrt{\varepsilon}$ up to the nearest integer, let that integer be n_0 .

Now we compute the limit of the sequence by separating z_n into its real and imaginary parts:

$$\lim_{n \rightarrow \infty} \left(-2 + i \frac{(-1)^n}{n^2} \right) = \left(\lim_{n \rightarrow \infty} -2 \right) + i \left(\lim_{n \rightarrow \infty} \frac{(-1)^n}{n^2} \right) = -2 + i0 = -2$$