

Math 417 – Sections 6 & 7 Solutions

1. (a) $z = \frac{i}{-2-2i} = \frac{z_1}{z_2}$

An argument for $z_1 = i$ is $\theta_1 = \frac{\pi}{2}$ and an argument for $z_2 = -2 - 2i$ is $-\frac{3\pi}{4}$. Using Equation (6) on p. 19, we have:

$$\arg\left(\frac{z_1}{z_2}\right) = \arg z_1 - \arg z_2 = \frac{\pi}{2} - \left(-\frac{3\pi}{4}\right) = \frac{5\pi}{4}$$

The principal argument must lie in the interval $(-\pi, \pi]$. Therefore, $\text{Arg } z = \frac{5\pi}{4} - 2\pi = -\frac{3\pi}{4}$.

(b) $z = (\sqrt{3} - i)^6$

An argument for $\sqrt{3} - i$ is $\theta = -\frac{\pi}{6}$. Therefore, an argument for z is $\theta = 6(-\frac{\pi}{6}) = -\pi$. The principal argument is then $\text{Arg } z = -\pi + 2\pi = \pi$.

2. (a) $|e^{i\theta}| = |\cos \theta + i \sin \theta| = \sqrt{\cos^2 \theta + \sin^2 \theta} = 1$

(b) $\overline{e^{i\theta}} = \overline{\cos \theta + i \sin \theta} = \cos \theta - i \sin \theta = \cos(-\theta) + i \sin(-\theta) = e^{-i\theta}$