

Math 417 – Section 81 Solutions

1. Note that on the semicircle C_R in Figure 103 on p. 288 ($\gamma > \sqrt{2}$),

$$|F(s)| = \left| \frac{s^3}{s^4 - 4} \right| \sim \frac{R^3}{R^4} = \frac{1}{R} \rightarrow 0 \quad \text{as } R \rightarrow \infty$$

The inverse Laplace Transform exists and its value is:

$$f(t) = \sum_{n=1}^4 \operatorname{Res}_{s=s_n} [e^{st} F(s)]$$

The residues occur at $s_{1,2} = \pm\sqrt{2}$ and $s_{3,4} = \pm i\sqrt{2}$. The values are:

$$\begin{aligned} \operatorname{Res}_{s=s_1} e^{st} F(s) &= \frac{e^{s_1 t} s_1^3}{(s_1 - s_2)(s_1 - s_3)(s_1 - s_4)} = \frac{1}{2} e^{\sqrt{2}t} \\ \operatorname{Res}_{s=s_2} e^{st} F(s) &= \frac{e^{s_2 t} s_2^3}{(s_2 - s_1)(s_2 - s_3)(s_2 - s_4)} = \frac{1}{2} e^{-\sqrt{2}t} \\ \operatorname{Res}_{s=s_3} e^{st} F(s) &= \frac{e^{s_3 t} s_3^3}{(s_3 - s_1)(s_3 - s_2)(s_3 - s_4)} = \frac{1}{2} e^{i\sqrt{2}t} \\ \operatorname{Res}_{s=s_4} e^{st} F(s) &= \frac{e^{s_4 t} s_4^3}{(s_4 - s_1)(s_4 - s_2)(s_4 - s_3)} = \frac{1}{2} e^{-i\sqrt{2}t} \end{aligned}$$

Therefore,

$$f(t) = \frac{e^{\sqrt{2}t} + e^{-\sqrt{2}t}}{2} + \frac{e^{i\sqrt{2}t} + e^{-i\sqrt{2}t}}{2} = \cosh(\sqrt{2}t) + \cos(\sqrt{2}t)$$

2. Note that on the semicircle C_R in Figure 103 on p. 288 ($\gamma > 0$),

$$|F(s)| = \left| \frac{2s - 2}{(s + 1)(s^2 + 2s + 5)} \right| \sim \frac{R}{R^3} = \frac{1}{R^2} \rightarrow 0 \quad \text{as } R \rightarrow \infty$$

The inverse Laplace Transform exists and its value is:

$$f(t) = \sum_{n=1}^3 \operatorname{Res}_{s=s_n} [e^{st} F(s)]$$

The residues occur at $s_1 = -1$ and $s_{2,3} = -1 \pm 2i$. The values are:

$$\begin{aligned} \operatorname{Res}_{s=s_1} e^{st} F(s) &= \frac{e^{s_1 t} (2s_1 - 2)}{s_1^2 + 2s_1 + 5} = -e^{-t} \\ \operatorname{Res}_{s=s_2} e^{st} F(s) &= \frac{e^{s_2 t} (2s_2 - 2)}{(s_2 + 1)(s_2 - s_3)} = \frac{1}{2} (1 - i) e^{(-1+2i)t} \\ \operatorname{Res}_{s=s_3} e^{st} F(s) &= \frac{e^{s_3 t} (2s_3 - 2)}{(s_3 + 1)(s_3 - s_2)} = \frac{1}{2} (1 + i) e^{(-1-2i)t} \end{aligned}$$

Therefore, after some simplification we have:

$$f(t) = e^{-t} (-1 + \cos 2t + \sin 2t)$$

3. Note that on the semicircle C_R in Figure 103 on p. 288 ($\gamma > 1$),

$$|F(s)| = \left| \frac{12}{s^3 + 8} \right| \sim \frac{1}{R^3} \rightarrow 0 \quad \text{as } R \rightarrow \infty$$

The inverse Laplace Transform exists and its value is:

$$f(t) = \sum_{n=1}^3 \operatorname{Res}_{s=s_n} [e^{st} F(s)]$$

The residues occur at $s_{1,2,3} = -2, 1 \pm i\sqrt{3}$. The values are:

$$\begin{aligned} \operatorname{Res}_{s=s_1} e^{st} F(s) &= \frac{12e^{s_1 t}}{(s_1 - s_2)(s_1 - s_3)} = e^{-2t} \\ \operatorname{Res}_{s=s_2} e^{st} F(s) &= \frac{12e^{s_2 t}}{(s_2 - s_1)(s_2 - s_3)} = -\frac{1}{2}(1 + i\sqrt{3})e^{(1+i\sqrt{3})t} \\ \operatorname{Res}_{s=s_3} e^{st} F(s) &= \frac{12e^{s_3 t}}{(s_3 - s_1)(s_3 - s_2)} = -\frac{1}{2}(1 - i\sqrt{3})e^{(1-i\sqrt{3})t} \end{aligned}$$

Therefore, after some simplification we have:

$$f(t) = e^{-2t} + e^t(\sqrt{3}\sin(\sqrt{3}t) - \cos(\sqrt{3}t))$$