

Math 417 – Sections 8 & 9 Solutions

1. (b) To find the square roots of $1 - \sqrt{3}i$, we first write the exponential form of the number. We have $r = 2$ and $\theta = -\frac{\pi}{3}$ so:

$$1 - \sqrt{3}i = 2 \exp \left[i \left(-\frac{\pi}{3} + 2k\pi \right) \right]$$

The square roots are of the form:

$$z^{1/2} = r^{1/2} e^{i\theta/2} = \sqrt{2} \exp \left[i \left(-\frac{\pi}{6} + k\pi \right) \right]$$

Taking $k = 0$ and $k = 1$ we have:

$$k = 0 : \quad z^{1/2} = \sqrt{2} \exp \left[i \left(-\frac{\pi}{6} \right) \right]$$

$$k = 1 : \quad z^{1/2} = \sqrt{2} \exp \left[i \left(\frac{5\pi}{6} \right) \right]$$

In rectangular form, the roots are:

$$z^{1/2} = \sqrt{2} \left[\cos \left(-\frac{\pi}{6} \right) + i \sin \left(-\frac{\pi}{6} \right) \right] = \frac{\sqrt{6}}{2} - i \frac{\sqrt{2}}{2}$$

$$z^{1/2} = \sqrt{2} \left[\cos \left(\frac{5\pi}{6} \right) + i \sin \left(\frac{5\pi}{6} \right) \right] = -\frac{\sqrt{6}}{2} + i \frac{\sqrt{2}}{2}$$

3. (a) To find the cube roots of -1 , we first write the exponential form of the number. We have $r = 1$ and $\theta = \pi$ so:

$$-1 = \exp [i (\pi + 2k\pi)]$$

The cube roots are of the form:

$$z^{1/3} = r^{1/3} e^{i\theta/3} = \exp \left[i \left(\frac{\pi}{3} + \frac{2k\pi}{3} \right) \right]$$

Taking $k = 0$, $k = 1$, and $k = 2$ we have:

$$k = 0 : \quad z^{1/3} = \exp \left[i \left(\frac{\pi}{3} \right) \right]$$

$$k = 1 : \quad z^{1/3} = \exp \left[i \left(\frac{3\pi}{3} \right) \right]$$

$$k = 2 : \quad z^{1/3} = \exp \left[i \left(\frac{5\pi}{3} \right) \right]$$

In rectangular form, the roots are:

$$z^{1/3} = \cos \left(\frac{\pi}{3} \right) + i \sin \left(\frac{\pi}{3} \right) = \frac{1}{2} + i \frac{\sqrt{3}}{2}$$

$$z^{1/3} = \cos \left(\frac{3\pi}{3} \right) + i \sin \left(\frac{3\pi}{3} \right) = -1$$

$$z^{1/3} = \cos \left(\frac{5\pi}{3} \right) + i \sin \left(\frac{5\pi}{3} \right) = \frac{1}{2} - i \frac{\sqrt{3}}{2}$$

