Math 320 – Linear Algebra – David Dumas – Fall 2018

Exam 1

Instructions:

- Solve **three** problems from the list below.
- Each answer should consist of a proof. Claims offered without proof will receive no credit.
- Unless the problem gives different instructions, the rules for writing proofs are the same as on the most recent homework, except that the only axioms and theorems you may cite are those on the attached sheet.
- When writing your solutions, clearly label each one with the problem number.

(1) Let V be a vector space over a field $\mathbb{F}$. Let $w,x,y,z \in V$ and $a \in \mathbb{F}$. Show directly from the vector space axioms that if

$$((aw+x)+ay)+z = \vec{0}$$

then

$$a(w+y) = -(x+z).$$

Justify each step in your proof using one of the vector space axioms. You are not permitted to use any theorems in your solution.

(2) Let $S = \{(1,1,0),(0,1,1),(1,0,1)\}$, a subset of $(\mathbb{Z}_2)^3$. Consider $(\mathbb{Z}_2)^3$ as a vector space over $\mathbb{Z}_2$.

(a) Is S linearly independent?
(b) Does S generate $(\mathbb{Z}_2)^3$?
(c) Is S a basis of $(\mathbb{Z}_2)^3$?
(d) What is the dimension of $\operatorname{span}(S)$?

(3) Let V be a vector space of dimension n over a field $\mathbb{F}$. Suppose that $\{v_1,\ldots,v_n\}$ generates V. Prove that $\{v_1,\ldots,v_n\}$ is linearly independent.

(4) Let W denote the set of all polynomials $p \in P_4(\mathbb{R})$ that satisfy $p(1) = 0$. Prove that W is a subspace of $P_4(\mathbb{R})$ and determine the dimension of W.