Math 445 – David Dumas – Spring 2019

Homework 4

Due Monday, February 18 in class (1:00pm)

(—) From the textbook: 19.2, 19.3, 19.7, 19.8, 20.3a

(P1) Consider the subspace topology on the set $\mathbb{Q}$ of rational numbers, as a subset of $\mathbb{R}$ with the standard topology. Here is one way to construct a continuous function from $\mathbb{Q}$ to $\mathbb{Q}$: Take a continuous function $f : \mathbb{R} \to \mathbb{R}$, restrict its domain to $\mathbb{Q}$, and if this restricted function takes only rational values, then we can restrict the codomain as well to obtain a continuous function $\mathbb{Q} \to \mathbb{Q}$. (For example, the function $f(x) = x^2 + x + 1$ on $\mathbb{R}$ yields a continuous function $\mathbb{Q} \to \mathbb{Q}$ in this way.)

Are *all* continuous functions $\mathbb{Q} \to \mathbb{Q}$ obtained in this way? (Whatever the answer, give a proof.)

Note: This assignment originally included another problem (20.4a), but that has been postponed to Homework 5 because we did not cover the uniform topology on $\mathbb{R}^{\mathbb{N}}$ before this homework was collected.