(1) **Pictures of inversions.** In each part of this problem, you are presented with a figure containing a distinguished circle $\gamma$ (the one drawn with a thicker line). Draw the result of applying the inversion in $\gamma$ to the entire figure.

For example, the following images show a pentagram and its inversion in the circumscribed circle:

Your drawings do not need to be “perfect” (i.e. you are not required to use a compass, nor to compute centers and radii analytically), but should accurately reflect the properties of inversions applied to lines, circles, angles, etc..
**Extra credit:** Draw the inversion of the following figure (which is a union of many circles) in the outermost circle. For this extra credit exercise, your figure will be evaluated based on a high standard of accuracy.

2

(2) **Asymptotically parallel lines.** Let \( P_+ = (0, 1), P_- = (0, -1), \) and \( Q = (1, 0). \) Let \( l_+ \) and \( l_- \) denote the lines in \( \Delta_K \) corresponding to the chords \( P_+Q \) and \( P_-Q, \) respectively. These lines are asymptotically parallel because they have a common ideal endpoint. In this problem you will use a calculation to justify this terminology.

Show that \( l_+ \) and \( l_- \) are asymptotic in the sense that for any \( \epsilon > 0, \) there are points \( X_+ \in l_+ \) and \( X_- \in l_- \) such that \( d_K(X_+, X_-) < \epsilon. \)

(Remember: In your solution you must assume you are given a small positive real number \( \epsilon, \) and then find the corresponding pair of points \( X_\pm. \) Your justification should include a calculation of \( d_K(X_+, X_-), \) and an argument showing it is less than \( \epsilon. \))

(3) **Ideal triangle calculations.** An ideal triangle in the hyperbolic plane is the figure formed by three lines, any two of which are asymptotically parallel. An ideal triangle is not a triangle, because it does not have any vertices. In the Klein model, an ideal triangle is a Euclidean triangle inscribed in the unit circle. Let \( T \) denote the ideal triangle in \( \Delta_K \) corresponding to an equilateral Euclidean triangle inscribed in \( S^1 \) with \((1, 0)\) as one of its vertices.

(a) Find the radius of the hyperbolic circle inscribed in \( T, \) i.e. the hyperbolic circle with center \((0, 0)\) that is tangent to each of the three edges of \( T. \) This radius is the inradius of \( T. \) (In your solution, include a picture of \( T \) and its inscribed circle.)

(b) Let \( t_1, t_2, t_3 \) be the three points of tangency of \( T \) and its inscribed circle. Then \( t_1, t_2, t_3 \) are the vertices of an equilateral triangle in \( \Delta_K. \) Find the side length of this triangle.

(4) **Coaxial pencils.** Complete exercise \textbf{P-13} and part (a) of exercise \textbf{P-14} from p. 284 of Greenberg. (Note: You can use the facts about radical axes from exercise P-7, including those which were not part of Homework 5. Also, when reading P-7, notice that Greenberg uses the notation \( XT \) to mean the distance between \( X \) and \( T, \) which is a real number. In lecture we have used the notation \( |XT| \) for this quantity.)

(5) **Pythagorean counterexample.** Show that the pythagorean theorem does not hold in hyperbolic geometry by finding an explicit counterexample. (In other words, calculate the side lengths \( a, b, c \) of a right triangle and show that they do not satisfy \( a^2 + b^2 = c^2. \))