

Homework 3

Due Friday February 8

[Recommended practice problems (do not turn these in): 3.7, 3.8, 4.4]

Problems from the textbook:

- Page 54 problems 7, 8, 9, 10, 11, 12.

Additional problems:

- (none this time)

Instructions for writing these proofs:

The short version: Reading the proofs in chapters 3–5 of the textbook should give you an idea of what is expected and allowed in these problems.

The details: These problems ask you to prove things about *integers* and about *real numbers*. We are no longer working in the setting of a general ordered field, and so you will not be referring to the list of ordered field axioms at all.

Each step in the proofs you write for this assignment should involve either a logical deduction (e.g. $P \implies Q$ and $Q \implies R$ therefore $P \implies R$, or replacing a statement with its contrapositive) or a manipulation of equalities or inequalities of integers or real numbers according to basic rules of algebra. Refer to the proofs in chapters 3–5 of the textbook for examples.

In particular, your proofs may use:

- Any of the properties of arithmetic listed on pages 19–20 of the textbook. You do *not* need to list the number of the property or properties you are using.
- Any of the axioms for inequalities that are listed on page 24 of the textbook. You do *not* need to list the number of the axiom or axioms you are applying.
- Any of the propositions about real numbers or integers that are stated and proved in the textbook in chapter 3 or 4. You do *not* need to give the proposition number. For reference, the ones that are general enough to be potentially useful include:
 - Prop. 3.1.1: For positive real numbers a and b , $a < b \implies a^2 < b^2$.
 - Prop. 3.1.4: For non-zero real numbers a , $a^2 > 0$.
 - Prop. 3.2.1: For real numbers a and b , $a < b \implies 4ab < (a + b)^2$.
 - Prop. 4.2.1: If a, b, c are integers such that $a > b$, then $a \leq b \implies c \leq 0$.
 - Prop. 4.4.1: For real numbers a and b , $ab = 0 \iff (a = 0) \text{ or } (b = 0)$.
 - Prop. 4.4.2 and 4.4.3, which give the rest of the conditions for determining how the sign of ab relates to those of a and b .