

Math 535
Homework 4
Due Friday, February 20

Begin Chapter 4. You are encouraged to work on *all* of the exercises in the text, but you only need to turn in the following problems.

1. Find a conformal map taking the crescent-shaped region

$$\{z \in \mathbb{C} : |z| < 1 \text{ and } |z - 1| > \sqrt{2}\}$$

homeomorphically to the unit disk $\mathbb{D} = \{|z| < 1\}$. **Hint:** compute the angle where the boundary circles intersect.

2. Find a conformal covering map

$$f : \mathbb{D} \rightarrow A_R = \{z \in \mathbb{C} : 1 < |z| < R\}.$$

Give a formula for the 1-parameter family of conformal homeomorphisms $\varphi_\theta : \mathbb{D} \rightarrow \mathbb{D}$, $\theta \in \mathbb{R}$, such that $f(\varphi_\theta(z)) = e^{i\theta} f(z)$, with φ_0 the identity. (Thus, we have described a universal covering f of the annulus A_R , together with its deck transformations $\varphi_{2\pi n}$, $n \in \mathbb{Z}$.)

3. Ahlfors §1.3 #2, p.108, computing the integral of $x dz$ in two ways.
4. Ahlfors §1.3 #6, p.108, computing the integral of $f'(z)/f(z)$.
5. Ahlfors §2.1 #2, p.117, defining winding number for continuous curves.
6. Ahlfors §2.1 #3, p.118, Jordan curve theorem.