

Math 546
Topics in analysis: Complex dynamics
Course plan

Professor: Laura DeMarco

Course assistant: Hexi Ye

The goal of this course is to learn about holomorphic dynamics and the techniques from different areas of mathematics as they have been applied to study these systems. We will learn a sampling of methods from complex analysis, potential theory, ergodic theory, number theory and algebraic geometry, each put to use to solve a particular dynamical problem.

The course will have four parts:

- (1) Polynomial iteration. We'll study the basics of complex dynamics, concentrating on the family of quadratic polynomials $f(z) = z^2 + c$, for any complex number c , leading up to a discussion of the major open problem on density of hyperbolicity.
- (2) Advanced methods. I will introduce conformal invariants (e.g. moduli, harmonic measure) and combinatorial models (e.g. Yoccoz puzzles, Hubbard trees) which have been used to classify polynomial dynamical systems.
- (3) Arithmetic dynamics. We will look at the dynamics of polynomials where the base field is equipped with a non-archimedean absolute value. The questions are of a different nature, and we will see techniques combining traditional complex potential theory and its number-theoretic analogs.
- (4) Complex dynamics in higher dimensions. We will see a combination of tools from algebraic geometry and analysis in several complex variables used to analyze mappings on algebraic surfaces.

Homework. I will distribute homework problems and readings with some regularity. You will not need to submit your solutions. If you have questions, you should speak with me or meet with Hexi Ye during his office hours.

Projects. Each of you should choose a project related to the theme of the course. You will submit a short (typed) summary of your research and present your findings to the class. I will give you guidance; you may concentrate on something close to your own area of research if you wish.

Recommended textbooks

J. Milnor. *Dynamics in One Complex Variable*

L. Carleson and T. Gamelin. *Complex Dynamics*

J. Silverman. *The Arithmetic of Dynamical Systems*