# **Problems**

All problems are available in MyFinanceLab. An asterisk (\*) indicates problems with a higher level of difficults.

## The Expected Return of a Portfolio

- You are considering how to invest part of your retirement savings. You have decided to put \$200,000 into three stocks: 50% of the money in GoldFinger (currently \$25/share), 25% of the money in Moosehead (currently \$80/share), and the remainder in Venture Associates (currently \$2/share). If GoldFinger stock goes up to \$30/share, Moosehead stock drops to \$60/share, and Venture Associates stock rises to \$3 per share,
  - a. What is the new value of the portfolio?
  - b. What return did the portfolio earn?
  - c. If you don't buy or sell shares after the price change, what are your new portfolio weights?
- You own three stocks: 600 shares of Apple Computer, 10,000 shares of Cisco Systems, and 5000 shares of Colgate-Palmolive. The current share prices and expected returns of Apple, Cisco, and Colgate-Palmolive are, respectively, \$500, \$20, \$100 and 12%, 10%, 8%.
  - a. What are the portfolio weights of the three stocks in your portfolio?
  - b. What is the expected return of your portfolio?
  - c. Suppose the price of Apple stock goes up by \$25, Cisco rises by \$5, and Colgate-Palmolive falls by \$13. What are the new portfolio weights?
  - d. Assuming the stocks' expected returns remain the same, what is the expected return of the portfolio at the new prices?
- 3. Consider a world that only consists of the three stocks shown in the following table:

Stock	Total Number of Shares Outstanding	Current Price per Share	Expected Return
First Bank	100 Million	\$100	18%
Fast Mover	50 Million	\$120	12%
Funny Bone	200 Million	\$30	15%

- Calculate the total value of all shares outstanding currently.
- b. What fraction of the total value outstanding does each stock make up?
- c. You hold the market portfolio, that is, you have picked portfolio weights equal to the answer to part b (that is, each stock's weight is equal to its contribution to the fraction of the total value of all stocks). What is the expected return of your portfolio?
- 4. There are two ways to calculate the expected return of a portfolio: either calculate the expected return using the value and dividend stream of the portfolio as a whole, or calculate the weighted average of the expected returns of the individual stocks that make up the portfolio. Which return is higher?

## The Volatility of a Two-Stock Portfolio



Using the data in the following table, estimate (a) the average return and volatility for each stock, (b) the covariance between the stocks, and (c) the correlation between these two stocks.

Year	2010	2011	2012	2013	2014	2015
Stock A	-10%	20%	5%	-5%	2%	9%
Stock B	21%	7%	30%	-3%	-8%	25%



- Use the data in Problem 5, consider a portfolio that maintains a 50% weight on stock A and a 50% weight on stock B.
  - a. What is the return each year of this portfolio?
  - b. Based on your results from part a, compute the average return and volatility of the portfolio.

- c. Show that (i) the average return of the portfolio is equal to the average of the average returns of the two stocks, and (ii) the volatility of the portfolio equals the same result as from the calculation in Eq. 11.9.
- Explain why the portfolio has a lower volatility than the average volatility of the two stocks.



7. Using your estimates from Problem 5, calculate the volatility (standard deviation) of a portfolio that is 70% invested in stock A and 30% invested in stock B.



- 8. Using the data from Table 11.3, what is the covariance between the stocks of Alaska Air and Southwest Airlines?
- 9. Suppose two stocks have a correlation of 1. If the first stock has an above average return this year, what is the probability that the second stock will have an above average return?
- 10. Arbor Systems and Gencore stocks both have a volatility of 40%. Compute the volatility of a portfolio with 50% invested in each stock if the correlation between the stocks is (a) +1, (b) 0.50, (c) 0, (d) -0.50, and (e) -1.0. In which cases is the volatility lower than that of the original stocks?
- Suppose Wesley Publishing's stock has a volatility of 60%, while Addison Printing's stock has a volatility of 30%. If the correlation between these stocks is 25%, what is the volatility of the following portfolios of Addison and Wesley: (a) 100% Addison, (b) 75% Addison and 25% Wesley, and (c) 50% Addison and 50% Wesley.
- 12. Suppose Avon and Nova stocks have volatilities of 50% and 25%, respectively, and they are perfectly negatively correlated. What portfolio of these two stocks has zero risk?



- 13. Suppose Tex stock has a volatility of 40%, and Mex stock has a volatility of 20%. If Tex and Mex are uncorrelated,
  - a. What portfolio of the two stocks has the same volatility as Mex alone?
  - b. What portfolio of the two stocks has the smallest possible volatility?

# The Volatility of a Large Portfolio

- 14. Using the data in Table 11.1,
  - a. Compute the annual returns for a portfolio with 25% invested in North Air, 25% invested in West Air, and 50% invested in Tex Oil.
  - b. What is the lowest annual return for your portfolio in part a? How does it compare with the lowest annual return of the individual stocks or portfolios in Table 11.1?



- X 15. Using the data from Table 11.3, what is the volatility of an equally weighted portfolio of Microsoft, Alaska Air, and Ford Motor stock?
  - Suppose the average stock has a volatility of 50%, and the correlation between pairs of stocks is 20%. Estimate the volatility of an equally weighted portfolio with (a) 1 stock, (b) 30 stocks, (c) 1000 stocks.
  - 17. What is the volatility (standard deviation) of an equally weighted portfolio of stocks within an industry in which the stocks have a volatility of 50% and a correlation of 40% as the portfolio becomes arbitrarily large?
  - Consider an equally weighted portfolio of stocks in which each stock has a volatility of 40%, and the correlation between each pair of stocks is 20%.
    - a. What is the volatility of the portfolio as the number of stocks becomes arbitrarily large?
    - b. What is the average correlation of each stock with this large portfolio?
  - Stock A has a volatility of 65% and a correlation of 10% with your current portfolio. Stock B has a volatility of 30% and a correlation of 25% with your current portfolio. You currently hold both stocks. Which will increase the volatility of your portfolio: (i) selling a small amount of stock B and investing the proceeds in stock A, or (ii) selling a small amount of stock A and investing the proceeds in stock B?

- 20. You currently hold a portfolio of three stocks, Delta, Gamma, and Omega. Delta has a volatility of 60%, Gamma has a volatility of 30%, and Omega has a volatility of 20%. Suppose you invest 50% of your money in Delta, and 25% each in Gamma and Omega.
  - a. What is the highest possible volatility of your portfolio?
  - b. If your portfolio has the volatility in (a), what can you conclude about the correlation between Delta and Omega?

## Risk Versus Return: Choosing an Efficient Portfolio

- 21. Suppose Ford Motor stock has an expected return of 20% and a volatility of 40%, and Molson-Coors Brewing has an expected return of 10% and a volatility of 30%. If the two stocks are uncorrelated.
  - a. What is the expected return and volatility of an equally weighted portfolio of the two stocks?
  - b. Given your answer to part a, is investing all of your money in Molson-Coors stock an efficient portfolio of these two stocks?
  - c. Is investing all of your money in Ford Motor an efficient portfolio of these two stocks?
- 22. Suppose Intel's stock has an expected return of 26% and a volatility of 50%, while Coca-Cola's has an expected return of 6% and volatility of 25%. If these two stocks were perfectly negatively correlated (i.e., their correlation coefficient is -1),
  - Calculate the portfolio weights that remove all risk.
  - b. If there are no arbitrage opportunities, what is the risk-free rate of interest in this economy?

For Problems 23–26, suppose Johnson & Johnson and Walgreens Boots Alliance have expected returns and volatilities shown below, with a correlation of 22%.

	Expected Return	Standard Deviation
Johnson & Johnson	7%	16%
Walgreens Boots Alliance	10%	20%

- 23. Calculate (a) the expected return and (b) the volatility (standard deviation) of a portfolio that is equally invested in Johnson & Johnson's and Walgreens' stock.
- 24. For the portfolio in Problem 23, if the correlation between Johnson & Johnson's and Walgreens' stock were to increase,
  - a. Would the expected return of the portfolio rise or fall?
  - b. Would the volatility of the portfolio rise or fall?
- 25. Calculate (a) the expected return and (b) the volatility (standard deviation) of a portfolio that consists of a long position of \$10,000 in Johnson & Johnson and a short position of \$2000 in Walgreens.



- \*26. Using the same data as for Problem 23, calculate the expected return and the volatility (standard deviation) of a portfolio consisting of Johnson & Johnson's and Walgreens' stocks using a wide range of portfolio weights. Plot the expected return as a function of the portfolio volatility. Using your graph, identify the range of Johnson & Johnson's portfolio weights that yield efficient combinations of the two stocks, rounded to the nearest percentage point.
  - 27. A hedge fund has created a portfolio using just two stocks. It has shorted \$35,000,000 worth of Oracle stock and has purchased \$85,000,000 of Intel stock. The correlation between Oracle's and Intel's returns is 0.65. The expected returns and standard deviations of the two stocks are given in the table below:

	Expected Return	Standard Deviation	
Oracle	12.00%	45.00%	
Intel	14.50%	40.00%	

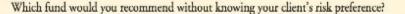
- a. What is the expected return of the hedge fund's portfolio?
- b. What is the standard deviation of the hedge fund's portfolio?

- 28. Consider the portfolio in Problem 27. Suppose the correlation between Intel and Oracle's stock increases, but nothing else changes. Would the portfolio be more or less risky with this change?
- \*29. Fred holds a portfolio with a 30% volatility. He decides to short sell a small amount of stock with a 40% volatility and use the proceeds to invest more in his portfolio. If this transaction reduces the risk of his portfolio, what is the minimum possible correlation between the stock he shorted and his original portfolio?
- 30. Suppose Target's stock has an expected return of 20% and a volatility of 40%, Hershey's stock has an expected return of 12% and a volatility of 30%, and these two stocks are uncorrelated.
  - a. What is the expected return and volatility of an equally weighted portfolio of the two stocks? Consider a new stock with an expected return of 16% and a volatility of 30%. Suppose this new stock is uncorrelated with Targer's and Hershey's stock.
  - b. Is holding this stock alone attractive compared to holding the portfolio in (a)?
  - Can you improve upon your portfolio in (a) by adding this new stock to your portfolio? Explain.
- 31. You have \$10,000 to invest. You decide to invest \$20,000 in Google and short sell \$10,000 worth of Yahoo! Google's expected return is 15% with a volatility of 30% and Yahoo!'s expected return is 12% with a volatility of 25%. The stocks have a correlation of 0.9. What is the expected return and volatility of the portfolio?
- 32. You expect HGH stock to have a 20% return next year and a 30% volatility. You have \$25,000 to invest, but plan to invest a total of \$50,000 in HGH, raising the additional \$25,000 by shorting either KBH or LWI stock. Both KBH and LWI have an expected return of 10% and a volatility of 20%. If KBH has a correlation of +0.5 with HGH, and LWI has a correlation of -0.50 with HGH, which stock should you short?

#### Risk-Free Saving and Borrowing

- \*33. Suppose you have \$100,000 in cash, and you decide to borrow another \$15,000 at a 4% interest rate to invest in the stock market. You invest the entire \$115,000 in a portfolio J with a 15% expected return and a 25% volatility.
  - a. What is the expected return and volatility (standard deviation) of your investment?
  - b. What is your realized return if J goes up 25% over the year?
  - c. What return do you realize if J falls by 20% over the year?
- 34. You have \$100,000 to invest. You choose to put \$150,000 into the market by borrowing \$50,000.
  - a. If the risk-free interest rate is 5% and the market expected return is 10%, what is the expected return of your investment?
  - b. If the market volatility is 15%, what is the volatility of your investment?
- 35. You currently have \$100,000 invested in a portfolio that has an expected return of 12% and a volatility of 8%. Suppose the risk-free rate is 5%, and there is another portfolio that has an expected return of 20% and a volatility of 12%.
  - a. What portfolio has a higher expected return than your portfolio but with the same volatility?
  - b. What portfolio has a lower volatility than your portfolio but with the same expected return?
- 36. Assume the risk-free rate is 4%. You are a financial advisor, and must choose one of the funds below to recommend to each of your clients. Whichever fund you recommend, your clients will then combine it with risk-free borrowing and lending depending on their desired level of risk.

	Expected Return	Volatility	
Fund A	10%	10%	
Fund B	15%	22%	
Fund C	6%	2%	





37. Assume all investors want to hold a portfolio that, for a given level of volatility, has the maximum possible expected return. Explain why, when a risk-free asset exists, all investors will choose to hold the same portfolio of risky stocks.

## The Efficient Portfolio and Required Returns

- 38. In addition to risk-free securities, you are currently invested in the Tanglewood Fund, a broad-based fund of stocks and other securities with an expected return of 12% and a volatility of 25%. Currently, the risk-free rate of interest is 4%. Your broker suggests that you add a venture capital fund to your current portfolio. The venture capital fund has an expected return of 20%, a volatility of 80%, and a correlation of 0.2 with the Tanglewood Fund. Calculate the required return and use it to decide whether you should add the venture capital fund to your portfolio.
- 39. You have noticed a market investment opportunity that, given your current portfolio, has an expected return that exceeds your required return. What can you conclude about your current portfolio?
- 40. The Optima Mutual Fund has an expected return of 20%, and a volatility of 20%. Optima claims that no other portfolio offers a higher Sharpe ratio. Suppose this claim is true, and the risk-free interest rate is 5%.
  - a. What is Optima's Sharpe Ratio?
  - b. If eBay's stock has a volatility of 40% and an expected return of 11%, what must be its correlation with the Optima Fund?
  - c. If the SubOptima Fund has a correlation of 80% with the Optima Fund, what is the Sharpe ratio of the SubOptima Fund?
- 41. You are currently only invested in the Natasha Fund (aside from risk-free securities). It has an expected return of 14% with a volatility of 20%. Currently, the risk-free rate of interest is 3.8%. Your broker suggests that you add Hannah Corporation to your portfolio. Hannah Corporation has an expected return of 20%, a volatility of 60%, and a correlation of 0 with the Natasha Fund.
  - a. Is your broker right?
  - b. You follow your broker's advice and make a substantial investment in Hannah stock so that, considering only your risky investments, 60% is in the Natasha Fund and 40% is in Hannah stock. When you tell your finance professor about your investment, he says that you made a mistake and should reduce your investment in Hannah. Is your finance professor right?
  - c. You decide to follow your finance professor's advice and reduce your exposure to Hannah. Now Hannah represents 15% of your risky portfolio, with the rest in the Natasha fund. Is this the correct amount of Hannah stock to hold?
- 42. Calculate the Sharpe ratio of each of the three portfolios in Problem 41. What portfolio weight in Hannah stock maximizes the Sharpe ratio?
- Returning to Problem 38, assume you follow your broker's advice and put 50% of your money in the venture fund.
  - a. What is the Sharpe ratio of the Tanglewood Fund?
  - b. What is the Sharpe ratio of your new portfolio?
  - c. What is the optimal fraction of your wealth to invest in the venture fund? (Hint: Use Excel and round your answer to two decimal places.)

## The Capital Asset Pricing Model

- 44. When the CAPM correctly prices risk, the market portfolio is an efficient portfolio. Explain why.
- 45. A big pharmaceutical company, DRIg, has just announced a potential cure for cancer. The stock price increased from \$5 to \$100 in one day. A friend calls to tell you that he owns DRIg.

You proudly reply that you do, too. Since you have been friends for some time, you know that he holds the market, as do you, and so you both are invested in this stock. Both of you care only about expected return and volatility. The risk-free rate is 3%, quoted as an APR based on a 365-day year. DRIg made up 0.2% of the market portfolio before the news announcement.

- a. On the announcement your overall wealth went up by 1% (assume all other price changes canceled out so that without DRIg, the market return would have been zero). How is your wealth invested?
- b. Your friend's wealth went up by 2%. How is he invested?
- 46. Your investment portfolio consists of \$15,000 invested in only one stock—Microsoft. Suppose the risk-free rate is 5%, Microsoft stock has an expected return of 12% and a volatility of 40%, and the market portfolio has an expected return of 10% and a volatility of 18%. Under the CAPM assumptions,
  - a. What alternative investment has the lowest possible volatility while having the same expected return as Microsoft? What is the volatility of this investment?
  - b. What investment has the highest possible expected return while having the same volatility as Microsoft? What is the expected return of this investment?
- 47. Suppose you group all the stocks in the world into two mutually exclusive portfolios (each stock is in only one portfolio): growth stocks and value stocks. Suppose the two portfolios have equal size (in terms of total value), a correlation of 0.5, and the following characteristics:

	Expected Return	Volatility
Value Stocks	13%	12%
Growth Stocks	17%	25%

The risk-free rate is 2%.

- a. What is the expected return and volatility of the market portfolio (which is a 50–50 combination of the two portfolios)?
- b. Does the CAPM hold in this economy? (Hint: Is the market portfolio efficient?)

#### **Determining the Risk Premium**

- 48. Suppose the risk-free return is 4% and the market portfolio has an expected return of 10% and a volatility of 16%. Merck & Co. (Ticker: MRK) stock has a 20% volatility and a correlation with the market of 0.06.
  - a. What is Merck's beta with respect to the market?
  - b. Under the CAPM assumptions, what is its expected return?



49. Consider a portfolio consisting of the following three stocks:

	Portfolio Weight	Volatility	Correlation with the Market Portfolio
HEC Corp	0.25	12%	0.4
Green Midget	0.35	25%	0.6
Alive And Well	0.4	13%	0.5

The volatility of the market portfolio is 10% and it has an expected return of 8%. The risk-free rate is 3%.

- a. Compute the beta and expected return of each stock.
- b. Using your answer from part a, calculate the expected return of the portfolio.
- c. What is the beta of the portfolio?
- d. Using your answer from part c, calculate the expected return of the portfolio and verify that it matches your answer to part b.