other. The idea of exchange options—options to exchange one asset for another—also will show up again in later chapters. We will see how to price such options in Chapter 14.

A second key idea that will prove important is the determination of factors influencing early exercise. As a practical matter, it is more work to price an American than a European option, so it is useful to know when this extra work is not necessary. Less obviously, the determinants of early exercise will play a key role in Chapter 17, where we discuss real options. We will see that certain kinds of investment projects are analogous to options, and the investment decision is like exercising an option. Thus, the early-exercise decision can have important consequences beyond the realm of financial options.

Much of the material in this chapter can be traced to Merton (1973b), which contains an exhaustive treatment of option properties that must hold if there is to be no arbitrage. Cox and Rubinstein (1985) also provides an excellent treatment of this material.

PROBLEMS

9.1 A stock currently sells for $32.00. A 6-month call option with a strike of $35.00 has a premium of $2.27. Assuming a 4% continuously compounded risk-free rate and a 6% continuous dividend yield, what is the price of the associated put option?

9.2 A stock currently sells for $32.00. A 6-month call option with a strike of $30.00 has a premium of $4.29, and a 6-month put with the same strike has a premium of $2.64. Assume a 4% continuously compounded risk-free rate. What is the present value of dividends payable over the next 6 months?

9.3 Suppose the S&R index is 800, the continuously compounded risk-free rate is 5%, and the dividend yield is 0%. A 1-year 815-strike European call costs $75 and a 1-year 815-strike European put costs $45. Consider the strategy of buying the stock, selling the 815-strike call, and buying the 815-strike put.

a. What is the rate of return on this position held until the expiration of the options?

b. What is the arbitrage implied by your answer to (a)?

c. What difference between the call and put prices would eliminate arbitrage?

d. What difference between the call and put prices eliminates arbitrage for strike prices of $780, $800, $820, and $840?

9.4 Suppose the exchange rate is 0.95 $/€, the euro-denominated continuously compounded interest rate is 4%, the dollar-denominated continuously compounded interest rate is 6%, and the price of a 1-year 0.93-strike European call on the euro is $0.0571. What is the price of a 0.93-strike European put?

9.5 The premium of a 100-strike yen-denominated put on the euro is ¥8.763. The current exchange rate is 95 ¥/€. What is the strike of the corresponding euro-denominated yen call, and what is its premium?

9.6 The price of a 6-month dollar-denominated call option on the euro with a $0.90 strike is $0.0404. The price of an otherwise equivalent put option is $0.0141. The annual continuously compounded dollar interest rate is 5%.

a. What is the 6-month dollar-euro forward price?
b. If the euro-denominated annual continuously compounded interest rate is 3.5%, what is the spot exchange rate?

9.7 Suppose the dollar-denominated interest rate is 5%, the yen-denominated interest rate is 1% (both rates are continuously compounded), the spot exchange rate is 0.009 $/¥, and the price of a dollar-denominated European call to buy one yen with 1 year to expiration and a strike price of $0.009 is $0.0006.

a. What is the dollar-denominated European yen put price such that there is no arbitrage opportunity?

b. Suppose that a dollar-denominated European yen put with a strike of $0.009 has a premium of $0.0004. Demonstrate the arbitrage.

c. Now suppose that you are in Tokyo, trading options that are denominated in yen rather than dollars. If the price of a dollar-denominated at-the-money yen call in the United States is $0.0006, what is the price of a yen-denominated at-the-money dollar call—an option giving the right to buy one dollar, denominated in yen—in Tokyo? What is the relationship of this answer to your answer to (a)? What is the price of the at-the-money dollar put?

9.8 Suppose call and put prices are given by

<table>
<thead>
<tr>
<th>Strike</th>
<th>50</th>
<th>55</th>
</tr>
</thead>
<tbody>
<tr>
<td>Call premium</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>Put premium</td>
<td>7</td>
<td>6</td>
</tr>
</tbody>
</table>

What no-arbitrage property is violated? What spread position would you use to effect arbitrage? Demonstrate that the spread position is an arbitrage.

9.9 Suppose call and put prices are given by

<table>
<thead>
<tr>
<th>Strike</th>
<th>50</th>
<th>55</th>
</tr>
</thead>
<tbody>
<tr>
<td>Call premium</td>
<td>16</td>
<td>10</td>
</tr>
<tr>
<td>Put premium</td>
<td>7</td>
<td>14</td>
</tr>
</tbody>
</table>

What no-arbitrage property is violated? What spread position would you use to effect arbitrage? Demonstrate that the spread position is an arbitrage.

9.10 Suppose call and put prices are given by

<table>
<thead>
<tr>
<th>Strike</th>
<th>50</th>
<th>55</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Call premium</td>
<td>18</td>
<td>14</td>
<td>9.50</td>
</tr>
<tr>
<td>Put premium</td>
<td>7</td>
<td>10.75</td>
<td>14.45</td>
</tr>
</tbody>
</table>

Find the convexity violations. What spread would you use to effect arbitrage? Demonstrate that the spread position is an arbitrage.

9.11 Suppose call and put prices are given by

<table>
<thead>
<tr>
<th>Strike</th>
<th>80</th>
<th>100</th>
<th>105</th>
</tr>
</thead>
<tbody>
<tr>
<td>Call premium</td>
<td>22</td>
<td>9</td>
<td>5</td>
</tr>
<tr>
<td>Put premium</td>
<td>4</td>
<td>21</td>
<td>24.80</td>
</tr>
</tbody>
</table>
Find the convexity violations. What spread would you use to effect arbitrage? Demonstrate that the spread position is an arbitrage.

9.12 In each case identify the arbitrage and demonstrate how you would make money by creating a table showing your payoff.

a. Consider two European options on the same stock with the same time to expiration. The 90-strike call costs $10 and the 95-strike call costs $4.

b. Now suppose these options have 2 years to expiration and the continuously compounded interest rate is 10%. The 90-strike call costs $10 and the 95-strike call costs $5.25. Show again that there is an arbitrage opportunity. (Hint: It is important in this case that the options are European.)

c. Suppose that a 90-strike European call sells for $15, a 100-strike call sells for $10, and a 105-strike call sells for $6. Show how you could use an asymmetric butterfly to profit from this arbitrage opportunity.

9.13 Suppose the interest rate is 0% and the stock of XYZ has a positive dividend yield. Is there any circumstance in which you would early-exercise an American XYZ call? Is there any circumstance in which you would early-exercise an American XYZ put? Explain.

9.14 In the following, suppose that neither stock pays a dividend.

a. Suppose you have a call option that permits you to receive one share of Apple by giving up one share of AOL. In what circumstance might you early-exercise this call?

b. Suppose you have a put option that permits you to give up one share of Apple, receiving one share of AOL. In what circumstance might you early-exercise this put? Would there be a loss from not early-exercising if Apple had a zero stock price?

c. Now suppose that Apple is expected to pay a dividend. Which of the above answers will change? Why?

9.15 The price of a non-dividend-paying stock is $100 and the continuously compounded risk-free rate is 5%. A 1-year European call option with a strike price of $100 \times e^{0.05 \times 1} = $105.127 has a premium of $11.924. A 1\frac{1}{2} year European call option with a strike price of $100 \times e^{0.05 \times 1.5} = $107.788 has a premium of $11.50. Demonstrate an arbitrage.

9.16 Suppose that to buy either a call or a put option you pay the quoted ask price, denoted \( C_a(K, T) \) and \( P_a(K, T) \), and to sell an option you receive the bid, \( C_b(K, T) \) and \( P_b(K, T) \). Similarly, the ask and bid prices for the stock are \( S_a \) and \( S_b \). Finally, suppose you can borrow at the rate \( r_B \) and lend at the rate \( r_L \). The stock pays no dividend. Find the bounds between which you cannot profitably perform a parity arbitrage.

9.17 In this problem we consider whether parity is violated by any of the option prices in Table 9.1. Suppose that you buy at the ask and sell at the bid, and that your continuously compounded lending rate is 0.3% and your borrowing rate is 0.4%. Ignore transaction costs on the stock, for which the price is $168.89. Assume that