Chapter 7: Straightening out the Curve

Introduction

Being a scientist is like being a detective. While scientists search for patterns in their data to solve the mysteries of nature, detectives search for patterns in their clues to solve the mysteries of the crime. We as scientists (you and your children) do experiments to find these patterns. Once found, a simple pattern will enable us to extrapolate and interpolate, in other words, to make predictions! Not all experiments will reveal patterns to us. For example, in *Arm Span vs. Height* the variables are only weakly correlated. The data points form a "cluster" rather than lying on a smooth curve.

The simplest pattern of all is when the data points lie on a straight line through (0,0). Straight lines are very easy to extrapolate. Just get a ruler. Even better, we can extrapolate to points beyond the last data point by using simple proportional reasoning. *The Bouncing Ball*, *Spreading Out II*, and *Circumference vs. Diameter* are just a few of the TIMS experiments that have graphs that are straight lines through (0,0). Of course, data never lie EXACTLY on a straight line because of measurement errors and the presence of uncontrolled variables. That's why we have to find the best-fit line.

When we study area and volume, we find some experiments that have a very definite pattern in the data, but the data points are definitely not on a straight line. What do we do then? Fortunately, Mother Nature is usually kind to us and her patterns are not too complicated.

Area vs. Perimeter

When we examine the data for *Area vs. Perimeter* (Figure 1a, 1b) we note that A and P are not proportional. The best fit curve for A vs. P does go through (0,0), but it is not a straight line either for the squares or for the equilateral triangles (See Figures 2a and 2b.). We could predict that just by looking at the data. Note that when we double P, the area more than doubles. In fact, it increases by a factor of four. When we triple P, the area increases by a factor of nine. This motivates us to apply an important mathematical problem solving technique: when you can't solve a problem, change it to a problem that you can solve. The fact that four is the square of two and nine is the

Squares

P in cm	A in sq cm
4	1
8	4
20	25

Figure 1a

Equilateral Triangles

P in cm	A in sq cm
6	2
12	7
24	28

Figure 1b

square of three leads us to guess that perhaps we should graph area versus the square of the perimeter. Wonder of wonders, when we do this the data points lie close to a straight line. (See Figures 3 and 4.) Now we have a more familiar situation, a straight line through (0,0). This is a problem we know how to handle. We can easily extrapolate, and we can use simple proportional reasoning.

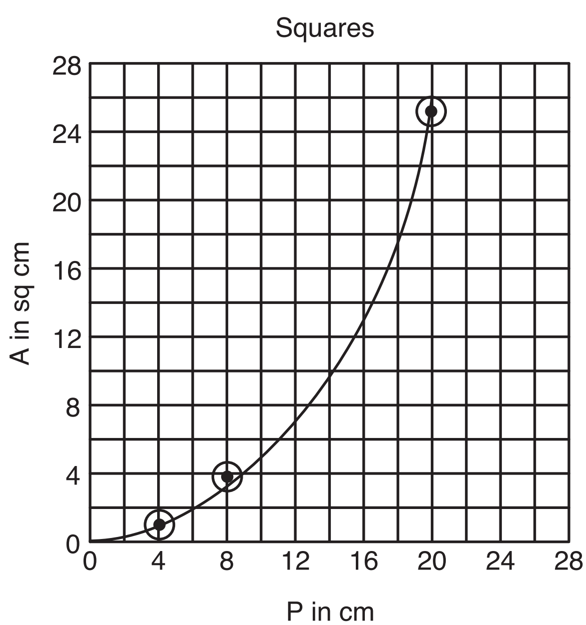

Figure 2a

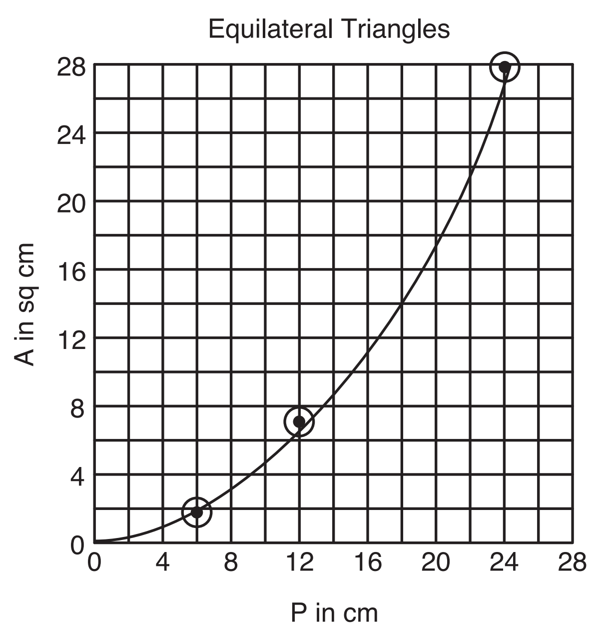

Figure 2b

Squares

P in cm	P^2 in sq cm	A in sq cm
4	16	1
8	64	4
20	400	25

Equilateral Triangles

P in cm	P^2 in sq cm	A in sq cm
6	36	2
12	144	7
24	576	28

Figure 3

The only difficulty is that our graph is A vs. $P^{2,}$ but we are interested not in P^2 but in P. This complicates our work. For example, consider the extrapolation question, What is the area of an equilateral triangle with a perimeter of 20cm? We can use the graph to answer this question, but first we must square 20 cm. So we look for the area of a triangle with a P^2 of 400 sq cm. By interpolating we find the area to be 20 sq cm (See Figure 4b).

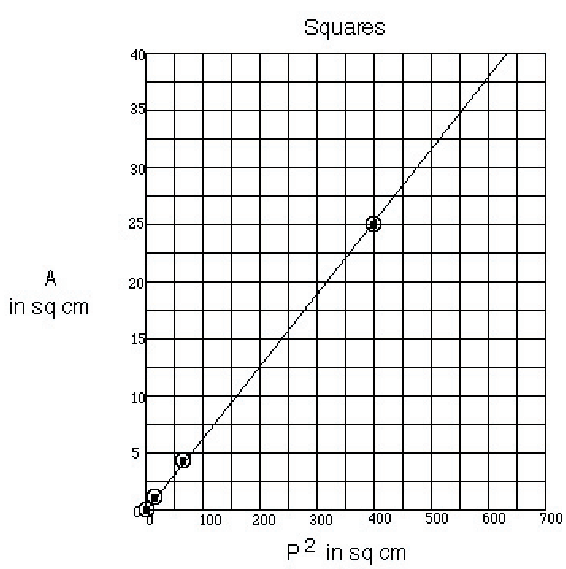

Figure 4a

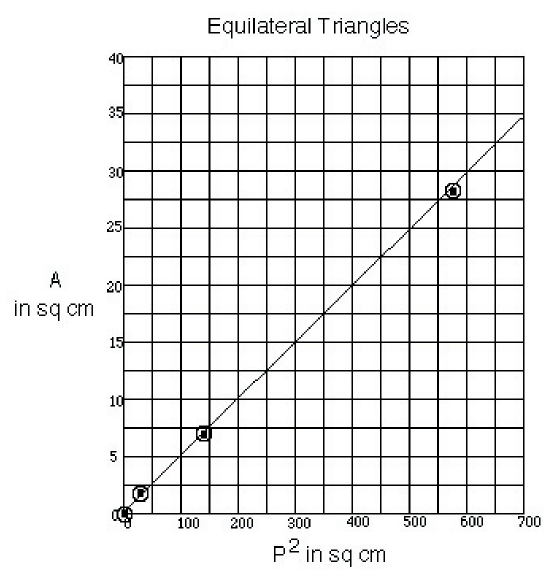

Figure 4b

We can also solve problems that lie beyond the curve by simple proportional reasoning. Since the curve is a straight line, the ratio of the vertical axis to the horizontal axis, A/P^2, is a constant! For example, what is the area of an equilateral triangle with a perimeter of 50 cm? Taking a nice point on the best-fit line in Figure 4b we find the ratio A/P^2 to be about (10 sq cm)/(200 sq cm). We have P = 50 cm which means $P^2 = 2500$ cm. So we must solve:

$$\frac{A}{P^2} = \frac{10 \text{ sq cm}}{200 \text{ sq cm}} = \frac{A}{2500 \text{ sq cm}}$$

$$A = \frac{(2500 \text{ sq cm})\,(10 \text{ sq cm})}{200 \text{ sq cm}}$$

$$A = 125 \text{ sq cm}$$

Solving for P when we know A is even a bit trickier. For example, What is the perimeter of an equilateral triangle with an area of 100 sq cm? Here we must solve:

$$\frac{A}{P^2} = \frac{10 \text{ sq cm}}{200 \text{ sq cm}} = \frac{100 \text{ sq cm}}{P^2}$$

$$P^2 = \frac{(200 \text{ sq cm})\,(100 \text{ sq cm})}{10 \text{ sq cm}}$$

$$P^2 = 2000 \text{ sq cm}$$

This is not quite what we want, however. We need to know P, not P^2. Here is a good chance to use a pocket calculator to find the square root f 2000 sq cm. In this way we find P is about 45 cm.

Thus, once we have straightened out the curve, we can use proportional reasoning to solve for unknown areas and perimeters, though there is an extra step because of the switch from P to P^2. This is a worthwhile mathematical extension of *Area vs. Perimeter* for your upper grade students.

Counting Out πR^2

We have designed *Counting Out* πR^2 to include straightening out the curve as an intrinsic part of the experiment. The reason is that we want the children to discover the very special and universal nature of the slope of the straight line. Typical data from *Counting Out* πR^2 are shown in Figure 5.

The variables are the radius of a circle R and its area A. If we graph the data, we see that it does not lie on a straight line (Figure 6). As before, when the

Counting Out πR^2

R in	A in sq cm
1.3	5.5
2.5	20
4.0	50

Figure 5

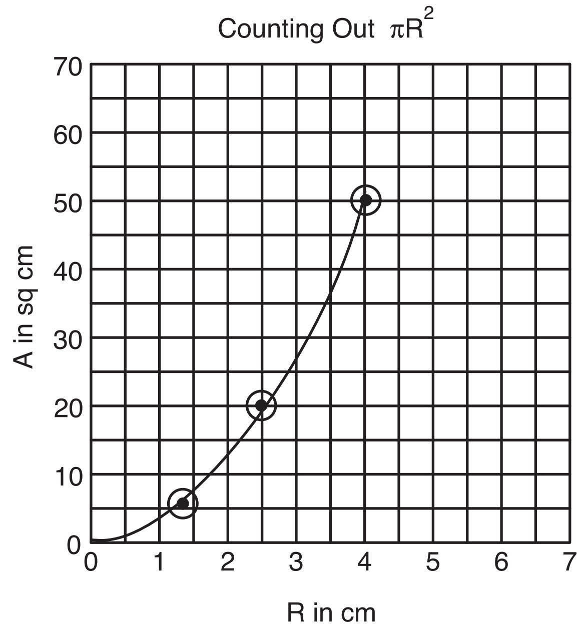

Figure 6

radius doubles, the area more than doubles. In fact, it increases by a factor of 4. When the radius triples, the area increases ninefold.

We have included a special table, shown in Figure 7, for the children to use to record their calculations of R^2 and the corresponding value of the area.

Their plot should look like Figure 8.

Since the plot of A vs. R^2 is a straight line through (0,0), we know the ratio A/R^2 must be a constant. Thus we have a simple pattern, which is easy to analyze using the techniques we have previously learned, providing we plot A vs. R^2 and not A vs. R. Using the curve we have $A/R^2 = 3.14$ (approximately). In fact, A/R^2, the slope of the line, is a famous number, π! Written as an equation we have the equally famous formula $A=\pi R^2$.

Note that we have proved $A=\pi R^2$ because both A and R were determined experimentally without using the formula.

Counting Out πR^2

R in cm	R^2 in $cm \times cm$	A in sq cm
1.3	1.7	5.5
2.5	6.25	20
4.0	16	50

Figure 7

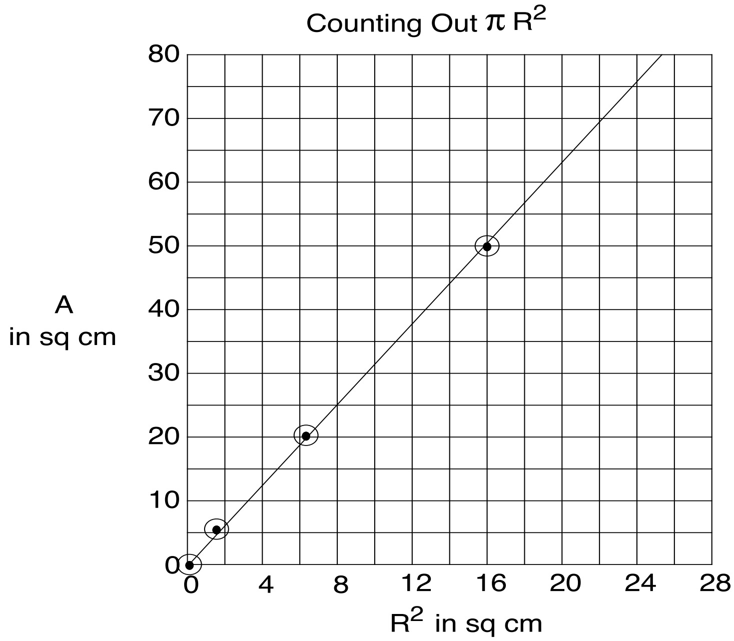

Figure 8

Cubes Count Too

Straightening out the curve is a general problem solving skill. For example, in the experiment *Volume vs. Diameter*, we plot the volume of spheres versus their diameter. (See Figure 9.) Once again the data do not lie on a straight line. If we try graphing v versus D^2, the curve is still not a straight line.

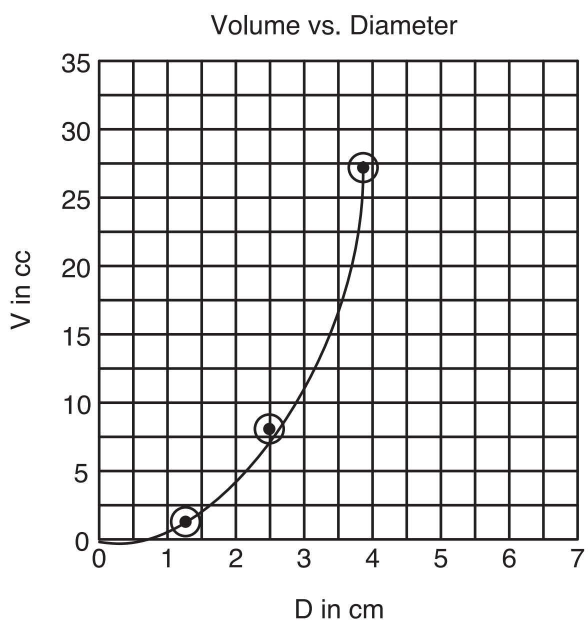

Figure 9

It is only when we graph volume versus D^3 that we finally get a straight line as shown in Figure 10. Thus we see that straightening out the curve may require squaring the linear dimension, or cubing it, or raising it to the 4^{th} power, or even raising it to a fractional power. Later we will see that this technique will help us understand the relationships between displacement and time for freely falling objects and between surface area and volume for solids of the same shape. If there is a simple relationship between the manipulated and responding variables, you will often find it by trying to straighten out the curve.

Table III

D in cm	D^3 in cc	V in cc
1.3	2.2	1
2.5	15.6	8
3.8	54.8	2.7

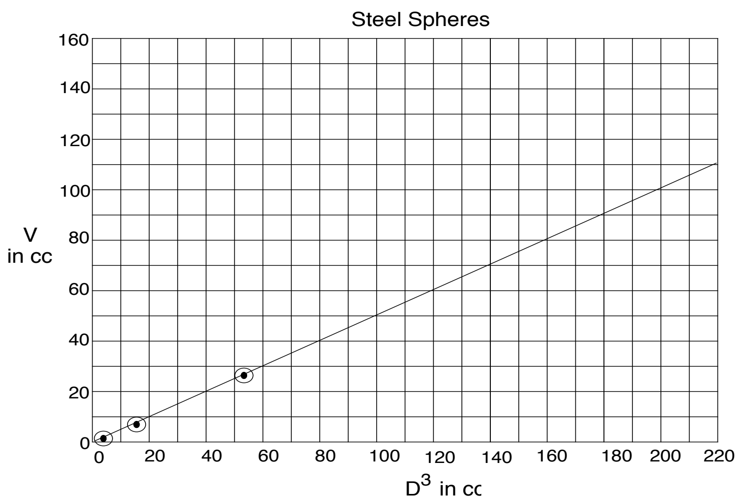

Figure 10

Units

The discovery of patterns becomes a game in which we guess at an expression involving the manipulated variable and see if graphing this against the responding variable gives us a straight line. Paying attention to the units of the manipulated and responding variables can often give us a clue for what to try. For example in *Counting Out* πR^2 both area and R^2 are measured in sq cm. Thus we might guess that area and R^2 are proportional.

Spreadsheets

The process of guessing an expression in the manipulated variable and graphing it against the responding variable is an effective one but very time consuming. The advent of the microcomputer graphing spreadsheet will offer us an opportunity to relieve some of the tedium of the trial and error method. Thus we can make a guess for an expression in the manipulated variable which predicts the responding variable. In an instant we can get a graphical representation and see whether we get a straight line or not.