Chapter 9: The Concept of Mass

Defining Mass

Mass is one of the most difficult variables to understand, but it is also one of the most important. As we have seen thus far, length, area, and volume have straightforward definitions. The volume of an object, for example, was defined as a measure of the space it occupies. Does mass have a similar straightforward definition? In a sense it does and in a sense it does not. Basically, mass is a measure of the quantity of matter in an object. That is the simple definition. Unfortunately, that begs the following question. One can ask, "What is matter?" and we are back to where we started. So we have to look more closely at the question of what exactly mass is. And that is where things get difficult. Mass is defined through what it does, and this sets it apart from length, area, and volume. Moreover, mass does two things, which means in a sense, it has two definitions.

One of the things that mass does is absolutely crucial for the existence of our universe. Mass is the cause of the force of gravity. Without mass, there would be no gravity, and without gravity, matter would not have clumped into galaxies and stars and us. Since mass causes gravity, we can define mass through the pull of gravity—the greater the pull, the greater the mass an object has.

The other thing that mass does has to do with the motion of an object. All other variables being equal, the mass of an object determines how much the velocity of an object will change when subject to a given force. In layman's terms, the larger the mass of an object, the harder you have to push to change its speed. Since we are not yet ready to study motion, we will not discuss the details of this aspect of mass. Nevertheless, we might note that it was Einstein's contemplation of both properties of mass (gravity and motion) that led him to his general theory of relativity.

As we consider the concepts of mass, gravity, and motion, we also need to know how to use to concepts to actually measure mass. One way, then, to determine which of two objects has more mass is to determine which one is pulled on more strongly by the Earth's gravitational force. One possible way to do this is to drop two objects, as shown in Figure 1, and see which reaches the ground first. You might suppose, since the more mass an object has, the bigger the force of gravity, so the more massive object would fall faster and reach the ground first. If you do this exercise, however (as Galileo did), you will find that both objects reach the ground at the same time! What went wrong? Well, we are breaking one of our cardinal rules, which is keeping variables fixed. When the objects fall, we not only have gravity pulling on them, but their velocity is changing as well. Therefore, both definitions of mass are involved and the overall effect is no longer obvious. It appears that in some way the effects cancel!

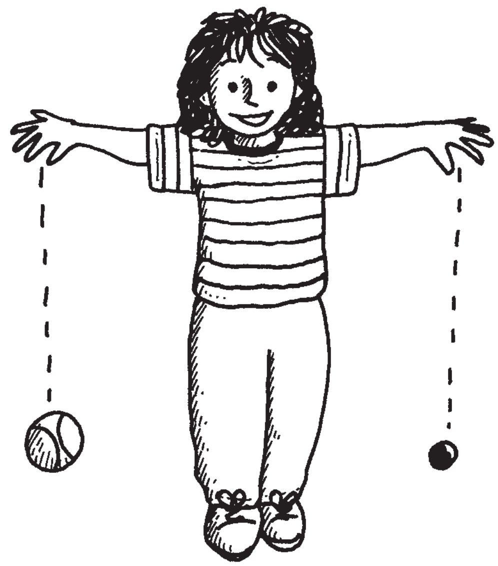

Figure 1: Dropping two objects

So we need a way of measuring the pull of gravity (i.e., using definition 1) without having the object move (i.e., eliminating definition 2). As we shall see in the next section, we can do this with an equal arm balance.

The Equal Arm Balance

The equal arm balance is the key to our operational definition of mass for children. If two objects balance, they have the same mass. If the arm tilts to one side, the object in the lower of the two pans has more mass than the object in the other pan.

Basically, the equal arm balance is a stick that is pivoted at its center and has two holders that are mounted on at points that are equidistant from the pivot (hence the name equal arm balance). Figure 2 pictures a balance that we use in Math Trailblazers. It is quite sturdy and accurate enough for use in the elementary classroom. The tall wooden shaft and long arms of the balance make it very useful in a wide variety of balancing experiments. If you would like to make your own, you can use a block of wood (Figure 3a) or a book

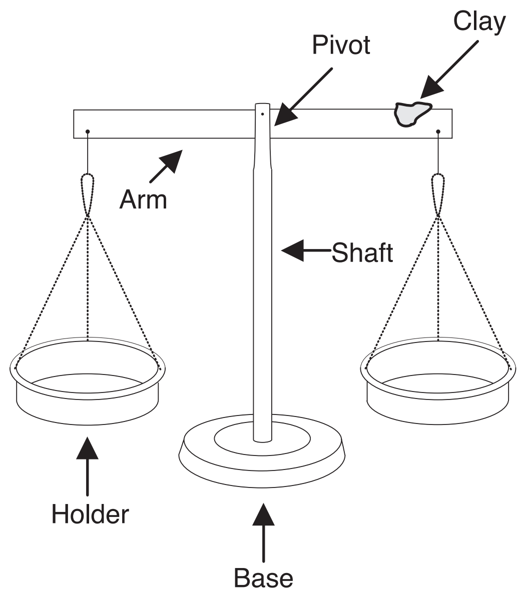

Figure 2:
An equal arm balance

to hold the pivot rod (Figure 3b). A ruler can act as the arm and the bottom of paper cups as pans. These latter pieces can be attached to the arm with string and paper clips.

Once you have put your equal arm balance together, it must be zeroed; that is, the arm must be level before any masses are added. You can do this by adding small pieces of clay to one end of the arm, as shown in Figure 2, until it is level. We suggest that you do this once for each balance and then have the children check to see that it is level before and during each experiment.

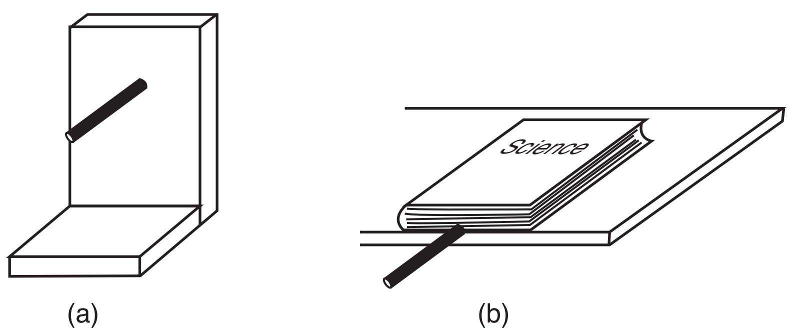

Figure 3:
Making a balance

We are now ready to find out which of two objects, A or B, has greater mass. One is placed in each pan. Since the object with more mass will experience the larger gravitational pull, the balance will tilt either one way or the other. If A has more mass it will tilt to the left, as shown in Figure 4, while if B has more mass it will tilt to the right. If $m_A = m_B$, the pull of gravity on both is the same and the balance will remain level.

One word of caution, though: If the masses are the same and the arm is tilted when the masses are placed on the balance, then the ruler will often remain tilted. Thus, to be sure that $m_A = m_B$, bring the arm back to level and see if it remains there. One of the first exercises you can do with children in kindergarten through second grade is to have them compare, two at a time, the masses of a wide variety of objects. Then they should order the objects from the most massive to the least massive. A washer, connecting cube, small scissors, steel ball, etc., can be used. Notice that the equal arm balance does away with the motion definition of mass since the balance is at rest when the measurement is made. Thus, the equal arm balance allows us to relate the object's mass directly to the pull of gravity.

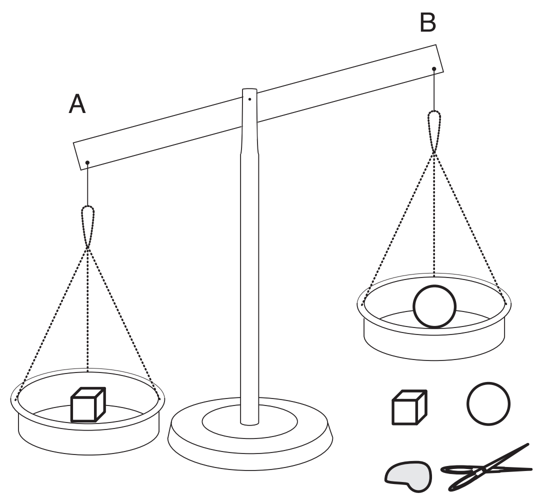

Figure 4:
Comparing masses

A	B	More Mass
Cube	Ball	Cube
Scissors	Ball	Scissors

Measuring Mass: The Mass Standard

The above is fine for comparing masses, but how do we measure the mass of an object? In other words, how do we assign a number to the mass of our object? Just as we did when we wanted to measure length, area, and volume, we need to decide on a unit. Say that we take as our standard masses a set of identical washers (paper clips would do as well). Let's call the mass of each washer 1 ugh. Then, if our object is balanced by 4 washers, its mass is 4 ughs, as shown in Figure 5. If the mass of the object is between 4 and 5 ughs, then the balance will not level out but tilt one way for 4 washers and the other way for 5. In this way, the child can assign unique masses to a wide range of objects. Clearly, the smaller the washer, the more accurately one can determine the mass of an object.

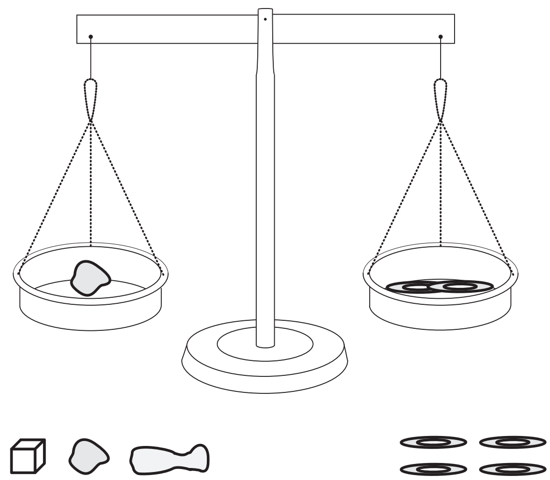

Figure 5:
Measuring masses

Object	Number of Washers	Mass in ughs
Ball	4	4
Cube	2	2

But washers are not a very satisfactory standard. The washers in your class may not have the same mass as those used in another school in your district, much less in another city or another country. What we need is a universal standard that is accepted by the entire scientific community. This problem was recognized by scientists a long time ago and was resolved when the Paris Academy of Sciences submitted a report to the French National Assembly in which 1 cc of water was defined to have a mass of 1 gm, the gram being the name chosen for the unit of mass. Then, using this definition, a platinum cylinder was made and declared to be the standard for 1000 grams. In 1875, an international treaty was signed by most “civilized” nations that established an International Bureau of Weights and Measures in Sevres, France, near Paris (the prototype meter is also kept there too, as you may recall). The international prototype kilogram, made of platinum-iridium alloy, is kept there. If you want your own kilogram, you have to go to Paris with an equal arm balance and some material and hack away at the material until it balances the platinum-iridium standard. The National Bureau of Standards in Washington, D.C., has an accurately constructed copy, as do other governments throughout the world.

For small measurements, one needs a mass that is smaller than a kilogram, just as one needs a length that is smaller than a meter. The gram, like the centimeter, is perfect for this. Most objects that you will deal with in the elementary school science program will have masses between 1 and 100 grams. Masses are commercially available, usually in 1-, 5-, 10-, 20-, 50-, 100-, 200-, 500-, and 1000-gram pieces. It might be a good idea to have one good set of very accurate standard masses for your school (locked in a closet marked "Paris," of course), but they are expensive. A complete set of brass or other metal masses can cost more than $80 (at 1994 prices). For general classroom use, there are less expensive sets of plastic masses available. A set of such masses is included in the Math Trailblazers manipulatives kit. However, since the plastic masses may not be as accurate, you may wish to compare them to a good set of standard masses or find their mass using a triple beam balance or other accurate scales.

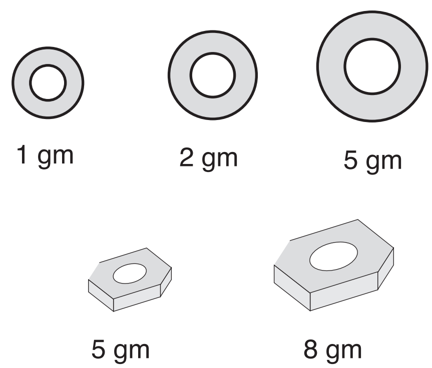

Figure 6:
Using washers as standard masses

Figure 7:
Using standard masses

Object	Number of Standard Masses				Mass in gm
	1 gm	2 gm	5 gm	8 gm	
Plastic Cylinder	3	1	2	2	31

The cheapest standard mass is a nickel. It has a mass very close to 5 grams! But alas, a dime does not have a mass of 10 grams. There are two ways to get around having to buy sets of expensive standard masses. To determine the mass of a small object, you can measure the mass of several of the objects and then divide the total mass by the number of objects. For example, if it turns out that 10 small objects balance a 5-gram piece, then each object will have a mass of 0.5 grams. (5 grams ÷ 10 objects = 0.5 grams per object.)

If you go to a hardware store and make a pest of yourself—bring your equal arm balance—you can usually find washers that have "nice" masses, like 1, 2, 5, 8 grams, etc. Figure 6 shows such a collection that can be used as standard masses. Once you have the washers, you can give each child several of each and a lot of objects whose mass you want them to determine. A good data table is a big help since you want to keep track of the number of different washers it takes to balance the object. Then, as shown in Figure 6, a little multiplication and addition is necessary to obtain the final mass in grams.

In the example from Figure 7, the total mass is:

$$m_{total} = 3 \text{ x } 1 \text{ gm} + 1 \text{ x } 2 \text{ gm} + 2 \text{ x } 5 \text{ gm} + 2 \text{ x } 8 \text{ gm} = 31 \text{ gm}.$$

Finding masses in grams is a great way to practice multiplication, addition, and mental math.

A second way to get around using expensive standards is again to use washers but place them in packages of sets of 1, 5, 10, 20 grams, etc. The package can be composed of several washers wrapped in masking tape with enough tape or small washers to make an even 5 gm or 10 gm, etc. Either way, the cost of a washer is at most 7 or 8 cents. The children, therefore, can have a set of masses for a few dollars.

Mass vs. Weight

In everyday language, we talk about the weight of an object rather than its mass. Are these two words for the same thing? Technically, weight and mass are distinct concepts. In scientific terms, the mass of an object is the amount of matter in the object. Mass is measured in kilograms and grams in the metric system and in pounds and ounces in the English system. The weight of an object is the measure of the pull of gravity on that object. Sir Isaac Newton first explained the importance of gravity for the motions of the planets and for the fall of an object more than 300 years ago.

Because of the awareness of space travel, most children know that the pull of gravity is different on different planets and that there is essentially no gravity in outer space. Many museums and planetariums have exhibits that show your weight on the moon and various planets. For example, since the moon's gravity is weaker than the Earth's, the pull on an individual object would be less. Thus, a human being would weigh less on the moon than on the Earth.

How is the mass of an object affected by gravity? The mass of an object remains constant regardless of space travel since gravity does not influence mass. Since we use a two-pan balance to measure mass, both sides of the balance are equally affected by gravity. If an 11-gram pencil balances one 1-gram and two 5-gram standard masses on Earth, it will balance those same masses on the moon. However, since most of us are likely to spend our lives on Earth, the distinction between mass and weight may be lost on many students.

We suggest two pedagogical alternatives with regard to the distinction between mass and weight:

1. Ignore it. This point is fairly subtle and eludes many adults.
2. Provide a simple explanation, but don't worry about it too much.