(1.)

a.) $A = \begin{array}{c} \\ \uparrow \\ \rightarrow \\ \downarrow \end{array} \begin{array}{c} \begin{array}{ccc} \uparrow & \rightarrow & \downarrow \end{array} \\ \begin{pmatrix} .5 & .25 & .45 \\ .3 & .1 & .15 \\ .2 & .65 & .4 \end{pmatrix} \end{array}$

b.) Thursday is 3 days later.

$$A^3 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0.44 \\ 0.21 \\ 0.36 \end{pmatrix}$$

It will rise 44% of the time.

c.) $x + y + z = 1$

$$\begin{pmatrix} .5 & .25 & .45 \\ .3 & .1 & .15 \\ .2 & .65 & .4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$\left(\begin{array}{ccc|c} -.5 & .25 & .45 & 0 \\ .3 & -.9 & .15 & 0 \\ .2 & .65 & -.6 & 0 \\ 1 & 1 & 1 & 1 \end{array}\right) \xrightarrow{\text{RREF}} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0.43 \\ 0 & 1 & 0 & 0.20 \\ 0 & 0 & 1 & 0.36 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

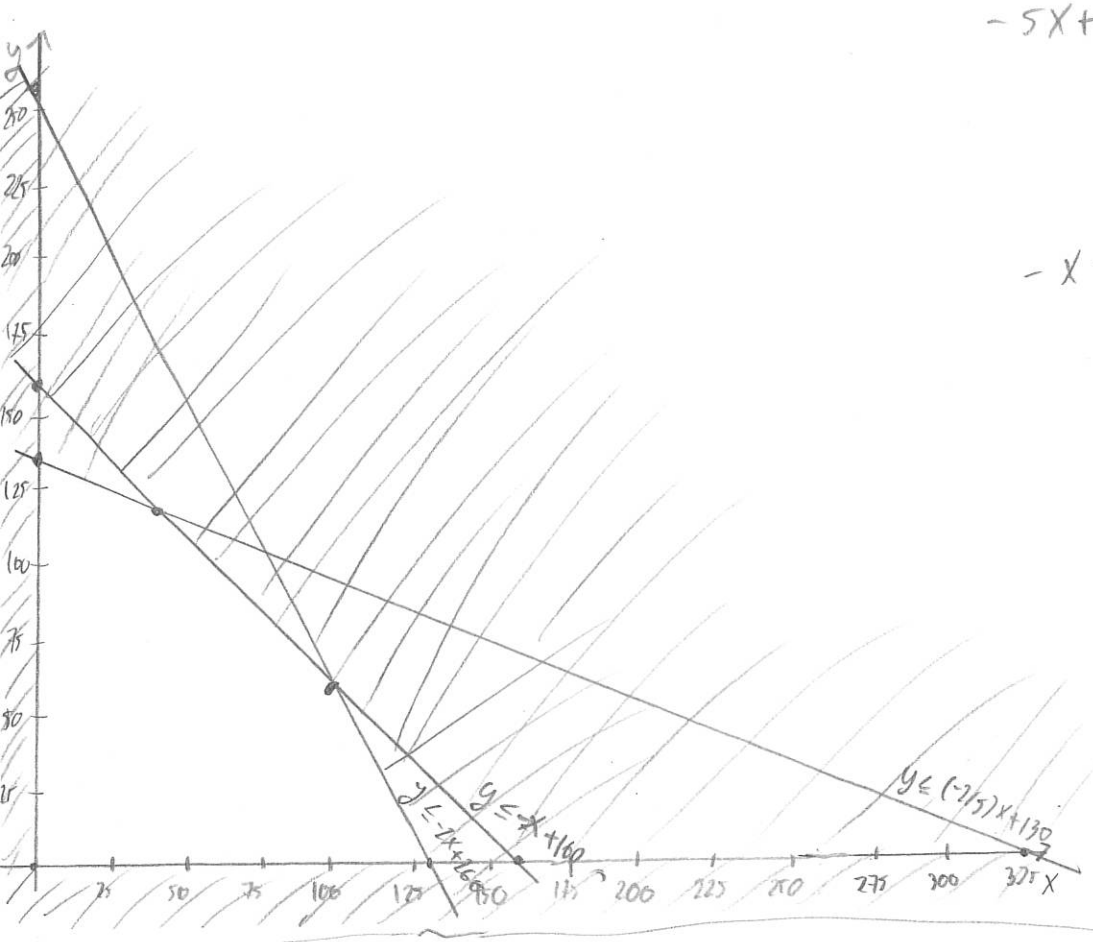
In the long term the stock will decrease 36% of the time.

(2.) maximize $7x+8y$

$$\begin{cases} y \le -x+160 & (0,160),(160,0) \\ y \le -2x+260 & (0,260),(130,0) \\ y \le (-2/5)x+130 & (0,130),(325,0) \\ x \ge 0, y \ge 0 \end{cases}$$

$$-x+160 = -\frac{2}{5}x+130$$
$$-5x+800 = -2x+650$$
$$3x = 150$$
$$x = 50$$
$$y = 110$$

$$-x+160 = -2x+260$$
$$x = 100$$
$$y = 60$$



x	y	7x+8y
50	110	1230
100	60	1180
0	0	0
0	130	1040
130	0	910

Max is 1230, attained at (50, 110).

$$\left(\begin{array}{cc|ccccc|c} 1 & \boxed{1} & 1 & 0 & 0 & 0 & 160 \\ 2 & 1 & 0 & 1 & 0 & 0 & 260 \\ 2 & \boxed{5} & 0 & 0 & 1 & 0 & 650 \\ \hline -7 & -8 & 0 & 0 & 0 & 1 & 0 \end{array}\right) \begin{array}{l} 160 \\ 260 \\ 130 \end{array}$$

$$\left(\begin{array}{cc|ccccc|c} \boxed{3/5} & 0 & 1 & 0 & -1/5 & 0 & 30 \\ 3/5 & 0 & 0 & 1 & -1/5 & 0 & 130 \\ 2/5 & 1 & 0 & 0 & 1/5 & 0 & 130 \\ \hline -19/5 & 0 & 0 & 0 & 8/5 & 1 & 1040 \end{array}\right) \begin{array}{l} 50 \\ 81.25 \\ 325 \end{array}$$

$$\left(\begin{array}{ccccc|c} 1 & 0 & 5/3 & 0 & -1/3 & 50 \\ 0 & 0 & -3/3 & 1 & 1/3 & 50 \\ 0 & 1 & -2/3 & 0 & 1/3 & 110 \\ \hline 0 & 0 & 19/3 & 0 & 1/3 & 1230 \end{array}\right)$$

$x = 50$

$y = 110$

$M = 1230$

(3.) maximize $1500-4x+6y$ subject to

$y \le -5x+360$ $(0,360),(72,0)$

$y \le (-1/3)x+60$ $(0,60),(180,0)$

$y \le -x+80$ $(0,80),(80,0)$

$x \ge 0, y \ge 0$

$-x+80 = -5x+360$

$4x = 280$

$x = 70$

$y = 10$

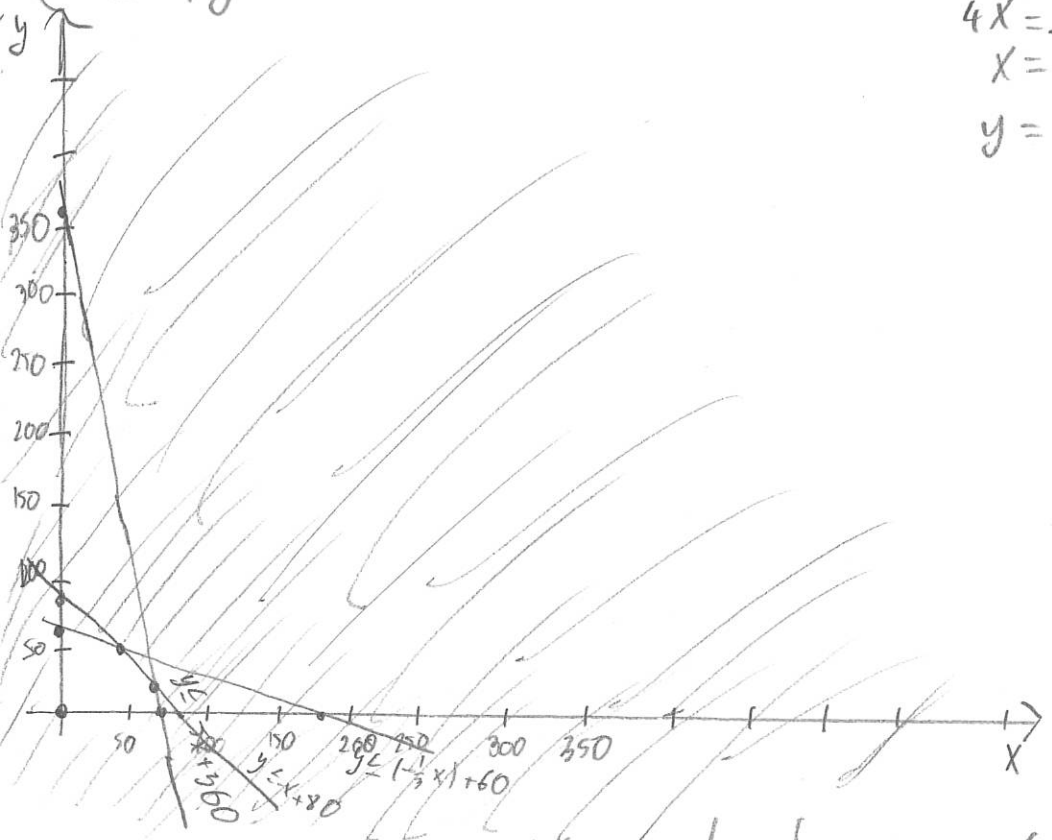
$-x+80 = -\frac{1}{3}x+60$

$-3x+240 = -x+180$

$2x = 60$

$x = 30$

$y = 50$



x	y	1500 - 4x + 6y
0	0	1500
0	60	1860
72	0	1212
70	10	1280
30	50	1680

Max is 1860, attained at $x=0, y=60$.

$$\left(\begin{array}{cccccc|c} 5 & 1 & 1 & 0 & 0 & 0 & 360 \\ 1 & 3 & 0 & 1 & 0 & 0 & 180 \\ 1 & 1 & 0 & 0 & 1 & 0 & 80 \\ \hline 4 & -6 & 0 & 0 & 0 & 1 & 1500 \end{array}\right) \begin{array}{c} 360 \\ 60 \\ 80 \end{array}$$

$$\left(\begin{array}{cccccc|c} 14/3 & 0 & 1 & -1/3 & 0 & 0 & 300 \\ 1/3 & 1 & 0 & 1/3 & 0 & 0 & 60 \\ 2/3 & 0 & 0 & -1/3 & 1 & 0 & 20 \\ \hline 6 & 0 & 0 & 2 & 0 & 1 & 1860 \end{array}\right)$$

$x = 0$

$y = 60$

$M = 1860$

(4.)

$$A = \begin{array}{c} \\ M \\ N \end{array}\begin{array}{c} \begin{array}{cc} M & N \end{array} \\ \begin{pmatrix} .4 & .7 \\ .6 & .3 \end{pmatrix} \end{array}$$

$x + y = 1$

$$\begin{pmatrix} .4 & .7 \\ .6 & .3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\left(\begin{array}{cc|c} 1 & 1 & 1 \\ -.6 & .7 & 0 \\ .6 & -.7 & 0 \end{array}\right) \xrightarrow{\text{RREF}} \left(\begin{array}{cc|c} 1 & 0 & .54 \\ 0 & 1 & .46 \\ 0 & 0 & 0 \end{array}\right)$$

The stable distribution is $\begin{pmatrix} .54 \\ .46 \end{pmatrix}$.

(5.) maximize $-2x+5y$ subject to

$$\begin{cases} -x+y \geq 40 \\ x \leq 60 \\ 3x+4y \leq 300 \\ x \geq 0,\ y \geq 0 \end{cases} \qquad \begin{cases} x-y \leq -40 \\ x \leq 60 \\ 3x+4y \leq 300 \\ x \geq 0,\ y \geq 0 \end{cases}$$

$$\left(\begin{array}{cccccc|c} 1 & \boxed{-1} & 1 & 0 & 0 & 0 & -40 \\ 1 & 0 & 0 & 1 & 0 & 0 & 60 \\ 3 & 4 & 0 & 0 & 1 & 0 & 300 \\ \hline 2 & -5 & 0 & 0 & 0 & 1 & 0 \end{array}\right) \begin{array}{l} 40 \\ \\ 300/4 = 75 \\ \\ \end{array} \longrightarrow \left(\begin{array}{cccccc|c} -1 & 1 & \boxed{1} & 0 & 0 & 0 & 40 \\ 1 & 0 & 0 & 1 & 0 & 0 & 60 \\ 7 & 0 & \boxed{4} & 0 & 1 & 0 & 140 \\ \hline -3 & 0 & -5 & 0 & 0 & 1 & 200 \end{array}\right) \begin{array}{l} -40 \\ \infty \\ 35 \\ \\ \end{array}$$

$$\left(\begin{array}{cccccc|c} 3/4 & 1 & 0 & 0 & 1/4 & 0 & 75 \\ 1 & 0 & 0 & 1 & 0 & 0 & 60 \\ 7/4 & 0 & 1 & 0 & 1/4 & 0 & 35 \\ \hline 23/4 & 0 & 0 & 0 & 5/4 & 1 & 375 \end{array}\right)$$

$x = 0$

$y = 75$

$M = 375$

(6.)

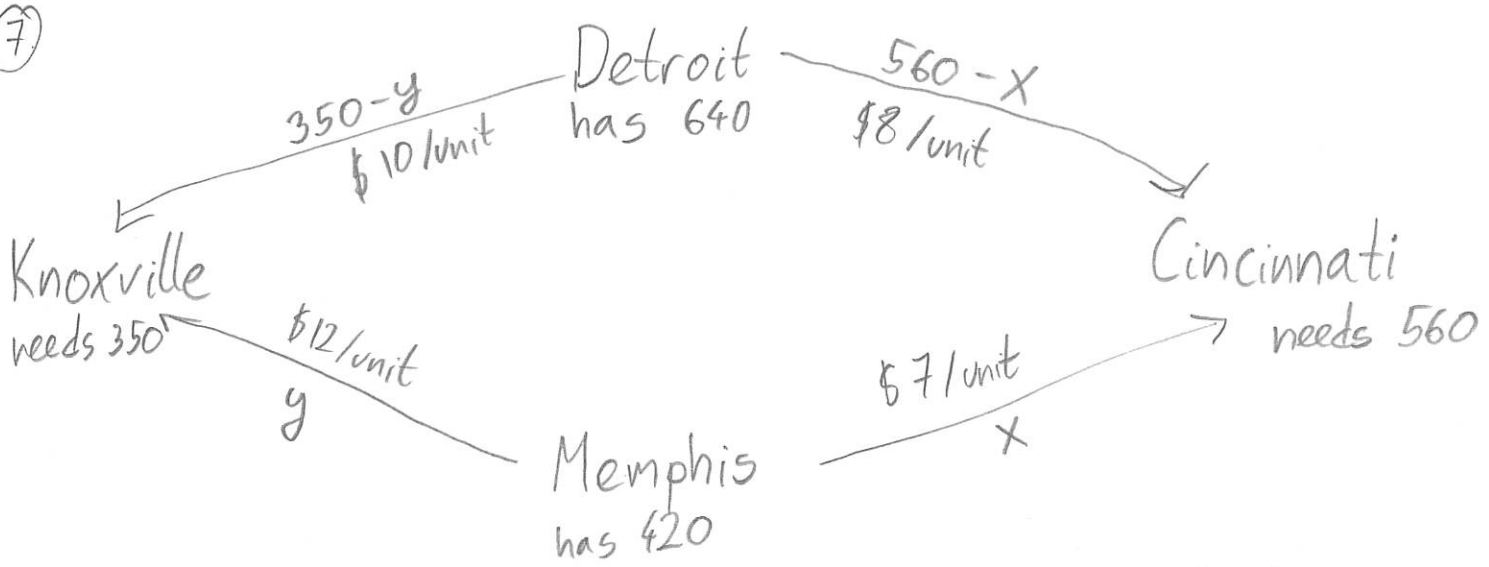
a.) $X_1 = 0$ $S_1 = 0$
$X_2 = 15$ $S_2 = 0$
$X_3 = 10$ $S_3 = 25$
$M = 125$

Another pivot necessary due to negative number in final row, column 1 (-2).

b.)
$X_1 = 0$ $S_1 = -15$
$X_2 = 0$ $S_2 = 0$
$X_3 = 10$ $S_3 = -2$
$M = 72$

Another pivot is necessary due to negative numbers in final column, rows 1 and 3.

(7)



minimize $7x + 8(560 - x) + 12y + 10(350 - y)$ subject to

$$\begin{cases} 560 - x + 350 - y \leq 640 \\ y + x \leq 420 \\ 350 - y \geq 0 \\ 560 - x \geq 0 \\ x \geq 0, y \geq 0 \end{cases}$$

$\downarrow\downarrow$

minimize $-x + 2y + 7980$ subject to

$$\begin{cases} x + y \geq 270 \\ x + y \leq 420 \\ y \leq 350 \\ x \leq 560 \\ x \geq 0, y \geq 0 \end{cases}$$

(8.)

$$A = \begin{array}{c} \\ DA \\ DD \end{array} \begin{array}{c} \begin{array}{cc} WA & WD \end{array} \\ \begin{pmatrix} .7 & .25 \\ .3 & .75 \end{pmatrix} \end{array}$$

a.) granddaughters are 2 generations away

$$\begin{pmatrix} .7 & .25 \\ .3 & .75 \end{pmatrix}^2 \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} .565 \\ .435 \end{pmatrix}$$

56.5% of granddaughters of women attending college will also attend.

b.) There seems to be a typo. This question is identical to a.).

(9.) x = # of chairs
y = # of tables

maximize $24x + 40y$ subject to

$$\begin{cases} 2x + 4y \leq 360 \\ x + 2y \leq 120 \\ x + 3y \leq 180 \\ 3y \geq x \\ x \geq 0, y \geq 0 \end{cases}$$

(10.)

a.) $A' = \begin{array}{c} \\ Q \\ R \\ P \\ S \end{array} \begin{array}{c} \begin{array}{cccc} Q & R & P & S \end{array} \\ \begin{pmatrix} 1 & 0 & .5 & .3 \\ 0 & 1 & .2 & .2 \\ 0 & 0 & .2 & .4 \\ 0 & 0 & .1 & .1 \end{pmatrix} \end{array}$

b.) $\begin{array}{c} \\ Q \\ R \\ P \\ S \end{array} \begin{array}{c} \begin{array}{cccc} Q & R & P & S \end{array} \\ \begin{pmatrix} 1 & 0 & .706 & .647 \\ 0 & 1 & .294 & .353 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$

$(I - R)^{-1} = \begin{array}{c} \\ P \\ S \end{array} \begin{array}{c} \begin{array}{cc} P & S \end{array} \\ \begin{pmatrix} 1.324 & .588 \\ .147 & 1.176 \end{pmatrix} \end{array}$

c.) 29.4% of objects starting in P will get absorbed in R.

d.) $.588 + 1.176 = 1.764$

On average it takes 1.764 cycles to get absorbed if starting in state S.

e.)
$$\begin{pmatrix} 1 & 0 & .706 & .647 \\ 0 & 1 & .294 & .353 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0.35 \\ 0.1 \\ 0.3 \\ 0.25 \end{pmatrix} = \begin{pmatrix} .729 \\ .276 \\ 0 \\ 0 \end{pmatrix}$$

Will end up in Q 72.9% of the time, in R 27.6%.

f.)
$$\begin{pmatrix} 1 & 0 & .706 & .647 \\ 0 & 1 & .294 & .353 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0.05 \\ 0.1 \\ 0.6 \\ 0.25 \end{pmatrix} = \begin{pmatrix} .635 \\ .365 \\ 0 \\ 0 \end{pmatrix}$$

Will end up in Q 63.5% of the time, in R 36.5%.

11.

$$A = \begin{array}{c} \\ H \\ L \\ N \end{array} \begin{array}{c} \begin{array}{ccc} H & L & N \end{array} \\ \begin{pmatrix} .3 & .35 & 1 \\ .5 & .4 & 0 \\ .2 & .25 & 0 \end{pmatrix} \end{array}$$

$$A^{100} = \begin{pmatrix} .446 & .446 & .446 \\ .372 & .372 & .372 \\ .182 & .182 & .182 \end{pmatrix}$$

In the long run 44.6% of students will be studying heavily.

(12.)

a.)

$32 - 34 = \div 2$

$2 - 2(-512) = 7$

$4 - 2(1) = 2$

$13 - 2(-112) = 14$

$47 - 2(112) = 46$

$x = 14$

$y = 0$

$z = 2$

$M = 46$

b.)

$32 - 30 = 2$

$2 + 2(1) = 4$

$4 + 2(-1) = 2$

$13 + 2(1) = 15$

$47 + 2(1) = 49$

$x = 15$

$y = 0$

$z = 2$

$M = 49$

c.)

$2 + h(1) \geq 0$

$4 + h(-1) \geq 0$

$13 + h(1) \geq 0$

$h \geq -2$

$h \leq 4$

$h \geq -13$

$-2 \leq h \leq 4$

(13)

minimize $14x+9y$ subject to

$$\begin{cases} x+y \geq 120 \\ -4x-y \geq -260 \\ 2x+5y \geq 600 \\ x \geq 0, y \geq 0 \end{cases}$$

$\downarrow\downarrow$ dual

maximize $120u-260v+600w$ subject to

$$\begin{cases} u-4v+2w \leq 14 \\ u-v+5w \leq 9 \\ u \geq 0, v \geq 0, w \geq 0 \end{cases}$$

(14.)

a.) maximize $60u + 8v$ subject to

$$\begin{cases} 10u + v \le 2 \\ 6u + v \le 3 \\ u \ge 0, v \ge 0 \end{cases}$$

b.)

	u	v	x	y	M	
	10	1	1	0	0 | 2	1/5
	6	1	0	1	0 | 3	1/2
	-60	-8	0	0	1 | 0	

c.)

	u	v	x	y	M	
	1	1/10	1/10	0	0 | 1/5	2
	0	2/5	-3/5	1	0 | 9/5	4.5
	0	-2	6	0	1 | 12	

↓

u	v	x	y	M
10	1	1	0	0 | 2
-4	0	-1	1	0 | 1
20	0	8	0	1 | 16

d.)

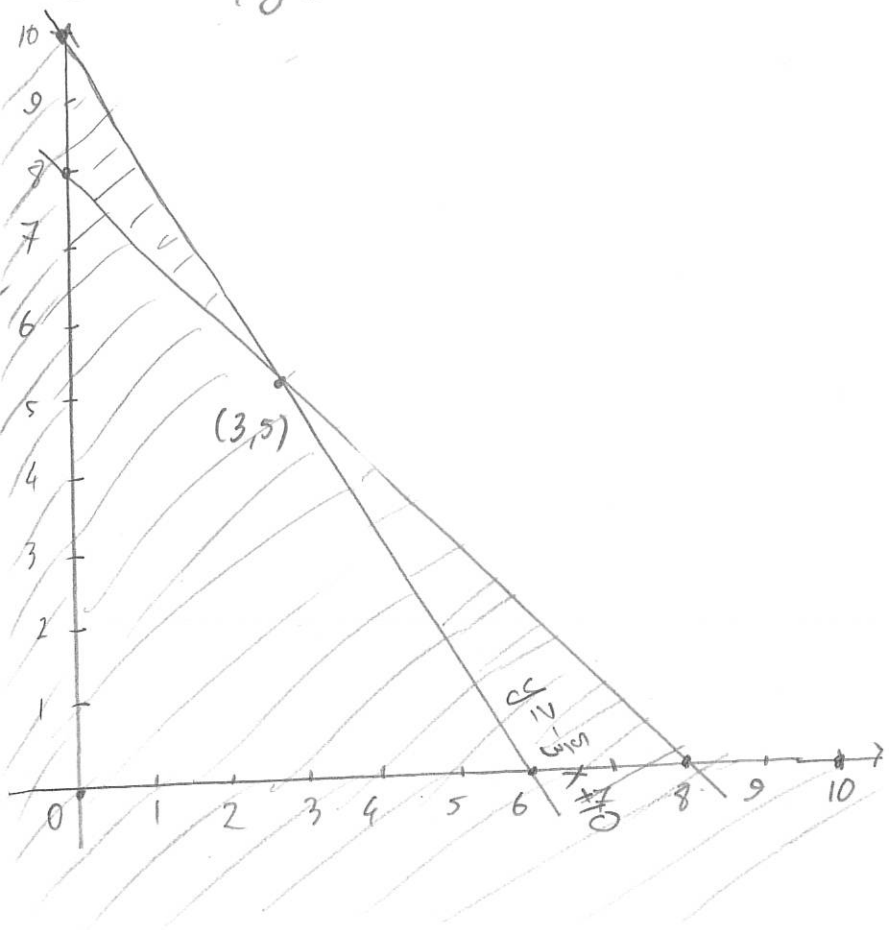
$x = 8$

$y = 0$

$M = 16$

15. Minimize $2x+3y$ subject to

$$\begin{cases} y \geq -\frac{5}{3}x+10 & (0,10),(6,0) \\ y \geq -x+8 & (0,8),(8,0) \\ x>0, y \geq 0 \end{cases}$$



$$-\frac{5}{3}x+10 = -x+8$$
$$-5x+30 = -3x+24$$
$$2x = 6$$
$$x = 3$$
$$y = 5$$

x	y	$2x+3y$
3	5	25
0	10	30
8	0	16

The minimum is 16 and attained at (8,0).

(16.)

x = # fancy homes

y = # family homes

$200 - x - y$ = # starter homes

maximize $4000(200-x-y) + 4500y + 8000x = 4000x + 500y + 800000$

$$\begin{cases} 10000(200-x-y) + 12000y + 18000x \le 3\,000\,000 \\ 60(200-x-y) + 75y + 80x \le 15000 \\ 200 - x - y \ge 0 \\ 200 - x - y + y \ge x \\ x \ge 0,\ y \ge 0 \end{cases}$$

$$\begin{cases} 8000x + 2000y \le 1000\,000 \\ 20x + 15y \le 3000 \\ x + y \le 200 \\ 2x \le 200 \\ x \ge 0,\ y \ge 0 \end{cases}$$

8000	2000	1	0	0	00	1000000	125
20	15	0	1	0	0 0	3000	150
1	1	0	0	1	0 0	200	200
(2)	0	0	0	0	1 0	200	100
-4000	-500	0	0	0	0 1	800000	

0	2000	1	0 0	-4000	0	200 000
0	(15)	0	1 0	-10	0	1000
0	1	0	0 1	-1/2	0	100
1	0	0	0 0	1/2	0	100
0	-500	0	0 0	2000	1	1200000

100
66.6(6)
100
∞

0	0	1	-400/3	0	-800/3	0	66666.6(6)
0	1	0	1/15	0	-2/3	0	200/3
0	0	0	-1/15	1	1/6	0	100/3
1	0	0	0	0	1/2	0	100
0	0	0	100/3	0	500/3	1	1233333.333

$x = 100$

$y = 200/3$

$200 - x - y = 100/3$

$M = 1233333.333$