Math 125 Final Exam Practice

1. List the Intercepts of the Inequality and then sketch the graph using the Math 125 method of shading

$$18x + 10y \geq 90$$

2. USING ALGEBRA(meaning no calculators), find the solution to the System of Equations.

$$4x - 5y = 16$$
$$6x + y = 7$$

3. Solve the Linear Programming Problem.
Graph the Inequalities and determine the Feasible Set. SHOW WORK.
Clearly mark your Axes and label graphed points with coordinates for full credit.

$$\textit{Maximize}: 1500 + 8x - 3y \quad \textit{subject to}$$
$$5x + y \leq 360$$
$$x + 3y \leq 180$$
$$x + y \leq 80$$
$$x \geq 0\ ,\ y \geq 0$$

4. DigginDeep Mining company has to fill an order for 40 tons of iron ore and 18 tons of cooper ore. They operate two mines producing both ores. Mine I yields four tons of iron and one ton of copper and costs $3200 per day to operate, while mine II yields two tons of iron and two tons of copper and costs $2600 per day to operate. Determine how many days each mine should be used to fill this order at the least total cost.
SET UP(do NOT solve) a Linear Programming Problem for this situation.

5. Solve each System of Equations using the "RREF" function on your calculator. Show your augmented matrix and the resulting one from RREF, plus your solution.

a) $$\begin{aligned} -x - 2y + 3z &= -14 \\ x + y - 8z &= 10 \\ x - y - 18z &= 3 \end{aligned}$$

b) $$\begin{aligned} x_1 + x_2 - x_3 &= 15 \\ 2x_1 + 3x_2 - 2x_3 &= 27 \\ -2x_1 - 3x_2 + 3x_3 &= -30 \end{aligned}$$

c) $$\begin{aligned} -4x - y + z &= -14 \\ 2x + y + z &= 6 \\ 5x + y - 2z &= 18 \end{aligned}$$

6. Write each Elementary Row Operation in notational values and then perform it on the matrix, using the original Matrix each time.

$$\begin{bmatrix} 3 & 2 & -2 & -6 & 5 \\ -3 & 12 & 7 & -2 & -6 \\ 0 & -4 & 5 & 1 & 3 \\ 1 & -2 & -3 & -7 & 5 \end{bmatrix}$$

a) Multiply Row 4 by positive three
b) Add minus 2 times row four to row one
c) Add 5 times row one to row three

7. Graph, BY HAND, the Feasible Set of the system below.

$$x + 2y \le 36$$
$$x + y \le 20$$
$$x \ge 0,\ y \ge 4$$

8. Given the following matrices, answer the questions below. You may use a calculator.

$$A = \begin{bmatrix} 2 & 3 \\ -4 & -5 \end{bmatrix} \quad B = \begin{bmatrix} 3 & 8 & 1 \\ 9 & -2 & -6 \\ -5 & 7 & 3 \end{bmatrix} \quad C = \begin{bmatrix} 2 & -5 & -6 \\ -1 & 4 & 9 \end{bmatrix} \quad D = \begin{bmatrix} 7 & -2 \\ 8 & 1 \\ -5 & -6 \end{bmatrix} \quad E = \begin{bmatrix} -4 & 8 & -1 \\ -5 & 3 & 7 \end{bmatrix}$$

a) Find: 2C – 3E
b) Find the product DE.
c) Is BDECA possible? Show why or why not

9. Pivot on the element in Row 3 and Column 2, BY HAND: $\begin{bmatrix} 2 & -3 & 4 & 1 \\ 0 & 5 & -2 & 7 \\ -8 & 2 & 4 & 6 \\ 1 & 4 & 6 & -4 \end{bmatrix}$

10. If $A = \begin{bmatrix} 13 & -18 \\ 6 & 17 \end{bmatrix}$ *and* $B = \begin{bmatrix} -10 & 7 \\ 16 & -23 \end{bmatrix}$, and 2A – C = B, find the matrix C.
You may use a calculator, but show any commands used.

11. A certain economy has three industries: metals, plastic, and wood. To produce $1 of output, the metals industry needs $0.08 of its own output, $0.03 from plastics, and $0.07 from wood. To produce $1 of output, the plastic industry needs $0.11 of its own output and $0.02 from wood. To produce $1 of output, the wood industry needs $0.10 of its own output, $.05 from metals, and $0.04 from plastic.
If the population needs(in Millions) $32 of metal, $25 of plastic, and $40 of wood, how much of each must be produced to satisfy this demand?
a) Name and show ALL relevant matrices
b) Use $X = (I - A)^{-1} D$ (calculator allowed) to determine production levels, rounded to the nearest tenth of a Million.

12. The Matrices below are Inverses of one another. Show ALL WORK in solving the system that follows,
USING the $X = A^{-1}B$ method.

$$\begin{bmatrix} 2 & 2 & 1 \\ -1 & 1 & -1 \\ 2 & 1 & 1 \end{bmatrix} \quad and \quad \begin{bmatrix} -2 & 1 & 3 \\ 1 & 0 & -1 \\ 3 & -2 & -4 \end{bmatrix}$$

$$-2x + y + 3z = 4$$
$$x - z = -2$$
$$3x - 2y - 4z = 8$$

13. A plane carries two types of packages. Type A weighs 140 pounds and takes up 3 cubic feet of space and generates $175 in revenue, while a Type B package weighs 185 pounds and takes up 5 cubic feet, generating revenues of $240 per package. The plane is limited to 6000 total pounds in 300 total cubic feet of space. Construct a full Linear Programming problem with an Objective function and ALL necessary constraints. DO NOT SOLVE.

14. The Tablette Corporation makes tablet computers at plants in Thailand and Malaysia. At the Thailand plant, unit costs are $79 and fixed costs are $9000, while in Malaysia unit costs are $71 and fixed costs are $9400.

a) If Tablette needs 600 tablets made, which plant does so at a lesser cost? JUSTIFY YOUR ANSWER
b) If 1000 tablets need to be made, but total costs at each plant must be equal, how many should be made at each of the plants?

15. A home appliance manufacturer has been selling a kitchen stove model several markets and wishes to enter new markets. Atlanta has 6.1 million people and they sold 13,286 stoves, Tampa has 2.8 million with 5,123 sold, Miami has 6.4 million with 17,522 sold, Charlotte(NC) has 2.5 million with 4,848 sold, and Greenville(SC) has 1.4 million with 3,613 sold.

a) Using Population figures in Millions and sales as they are given, use LinReg in constructing the Least Squares Line of Best Fit for this data set. **Round-off values to the nearest thousandth**.
b) Use your line to predict the sales generated by entering a market like Orlando with 2.9 million people.
c) What sized market does your model suggest is necessary to sell 20,000 units?

16. Use the graphing methods of Chapter 3 to solve the Linear Programming problem below.
SHOW ALL WORK FOR FULL CREDIT. Your calculator's graph IS NOT acceptable.

$Minimize: \ 900 + 5x + 7y \ \ subject \ to$

$2x + 4y \geq 64$

$6x + 2y \geq 72$

$x + 4y \geq 44$

$x \geq 0, \ y \geq 0$

17. A new wildlife refuge is being populated with animals, specifically lions and tigers and bears(Oh My!). The park manager knows he can fit exactly 150 animals total. His experience tells him to have at least twice as many lions as tigers. Area limitations suggest he acquire no more bears than tigers. To make sure the public is likely to see each species during their visit, he wants at least 25 of each. Lions cost $40/day to care for, tigers cost $30, and bears cost $35. Help the manager build a population meeting his needs at the lowest daily cost.

Set up the Linear Programming Problem using only TWO variables.
DO NOT SOLVE.

18. Furniture Factory produces tables, chairs, and desks. Each table needs 3 hours of carpentry, 1 hour of sanding, and 2 hours of staining, with a profit of $12.50. A chair needs 2 hours of carpentry, 4 hours of sanding, and 1 hour of staining, with a profit of $20. Desks need 1 hour of carpentry, 2 hours of sanding, and 3 hours of staining, with a profit of $16. There are 660 hours of carpentry, 740 hours of sanding, and 853 hours of staining available each week. The manager would like to make as much money as he can.

a) Construct the Linear Programming problem, including an Objective Function and ALL constraints
b) Build a fully labeled **Initial** Simplex Tableau. Indicate where and why you will pivot.
c) Pivot until you reach an Optimal Solution, and **show the final Tableau**. You may **skip** showing any intermediate Tableaux
d) State your solution in terms of the word problem above

19. Simplex Method and Duality.
a) Put the problem into "standard" Minimum form
b) Convert the problem into Dual Form
c) Build the Initial Simplex Tableau, indicate where/why you will Pivot. Perform **ONE Pivot only**.
d) Answer each question: (i) is the current solution Feasible? (ii) is the current solution Optimal?

$Minimize: 14x+9y \quad subject \ \ to$

$x+y \geq 120$

$4x+y \leq 260$

$2x+5y \geq 600$

$x \geq 0 \ , \ y \geq 0$

20. A Linear Programming problem and its Optimal Tableau are given below:
20pts

$Maximize: z = 3x+5y+2z \quad s.t.$

$2x+4y+2z \leq 34$

$3x+6y+4z \leq 57$

$2x+5y+z \leq 30$

$x \geq 0 \ , \ y \geq 0 \ , \ z \geq 0$

x	y	z	u	v	w	M	
0	1	0	$-5/2$	1	1	0	2
0	-1	1	1	0	-1	0	4
1	3	0	$-1/2$	0	1	0	13
0	2	0	$1/2$	0	1	1	47

a) If the RHS of Constraint **#1** were to be changed to 30, determine the New Optimal M-value and the value of each decision variable. **USE sensitivity analysis, NOT by redoing Simplex Method**

b) If the RHS of Constraint **#3** were to be changed to 32, determine the New Optimal M-value and the value of each decision variable. **USE sensitivity analysis, NOT by redoing Simplex Method**

c) If the RHS of Constraint **#1** were changed to $34+h$, find the range of h so that we are still feasible.

21. Business Math students are used in an experiment, where they are sent into a T-shaped maze once each day. If they turn left to exit the maze, they are given a slice of apple pie, while if they turn right to exit the maze, they are given a zap of electricity. It has been observed that the day after they turn left(apple pie!), they will turn left again the next day 55% of the time, and that the day after they turn right, they will turn right again the next day 75% of the time.

a) Build and label a transition matrix and initial distribution matrix
b) If on the first day, half of the students go each way, what percent will go right the next day?
c) USE ALGEBRA to determine the probability of students going each way in the long run. You may check with use of your calculators, but no credit will be given without an algebraic solution.

22.
$$\begin{aligned} 3x + 2y - 2z - 2w &= -2 \\ 2x + 2y - 4z - 2w &= 3 \\ -x \qquad - 2z + w &= 2 \\ -x - 2y + 6z \qquad &= -2 \end{aligned}$$

For the system of linear equations given above, find the value of X in a solution where Y = -2.5

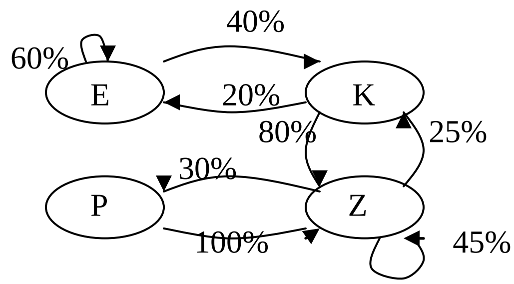
23. A small economy has three industries: coal, steel, and electricity. To produce $1 of coal, it takes $0.04 of coal, $0.20 of steel, and $.08 of electricity, while to produce $1 of steel, it takes $0.15 of coal and $0.10 of electricity. To produce $1 of electricity, it takes $0.02 of their own output, plus $0.30 of coal and $0.10 of steel. If the economy needs $54 million of coal, $26 of steel, and $85 of electricity, how much must be produced in the electricity industry, in millions to the nearest tenth?
(Hint: the useful formulas are $X - AX = D \qquad (I-A)^{-1}D = X$)

24. The initial absorbing matrix below has been converted to Standard form. Use $\left[\begin{array}{c|c} I & S(I-R)^{-1} \\ \hline 0 & 0 \end{array}\right]$ to determine what percentage of the population starting in Y will eventually end up in Z.

$$\begin{array}{c} \\ W \\ X \\ Y \\ Z \end{array}\begin{array}{c} \begin{array}{cccc} W & X & Y & Z \end{array} \\ \begin{bmatrix} .2 & 0 & .15 & 0 \\ .3 & 1 & .45 & 0 \\ .4 & 0 & .05 & 0 \\ .1 & 0 & .35 & 1 \end{bmatrix} \end{array} \Rightarrow \begin{array}{c} \\ X \\ Z \\ W \\ Y \end{array}\begin{array}{c} \begin{array}{cccc} X & Z & W & Y \end{array} \\ \begin{bmatrix} 1 & 0 & .3 & .45 \\ 0 & 1 & .1 & .35 \\ 0 & 0 & .2 & .15 \\ 0 & 0 & .4 & .05 \end{bmatrix} \end{array}$$

25. Given $A = \begin{bmatrix} 3 & -2 \\ -5 & 4 \end{bmatrix}$, $C = \begin{bmatrix} -6 & -2 \\ 11 & 6 \end{bmatrix}$ $\quad$ *and* $\quad AB = C$, find the total: $b_{2,1} + b_{2,2}$

26. Build a properly labeled Transition Matrix for the Transition Diagram below:



27.
$$\begin{aligned} &Minimize:\ 600-5x+7y \quad subject\ \ to: \\ &2x-3y \le 0 \\ &x+4y \le 132 \\ &x \ge 12 \end{aligned}$$

Solve by GRAPHING

28. Find the ***strictly determined*** solution of the game matrix below. **Show your reasoning**.

$$\begin{array}{c} \quad C \\ R \begin{bmatrix} 2 & 0 & 1 \\ 0 & -3 & 4 \\ 3 & -2 & 0 \end{bmatrix} \end{array}$$

29. Player R has \$1, \$10, and \$50 bills, while player C has \$5 and \$20 bills. Each randomly chooses a bill and shows it for each play of the game. The one with the larger bill collects the difference between their bill and that of the other player.

Write the payoff matrix for this game, showing any necessary labels on the matrix. Decide whether this game is Strictly Determined or not. If it is strictly determined, what is the Value of the game?

30. Below is a payoff matrix for a certain game. Do the requested tasks which follow.

$$\begin{array}{c} \quad C \\ R \begin{bmatrix} -15 & 40 & 25 \\ 25 & -10 & -5 \\ 45 & 20 & -15 \end{bmatrix} \end{array}$$

a) Show that the matrix does NOT have a strictly determined solution.

b) Find the expected payout if $R = \begin{bmatrix} .2 & .4 & .4 \end{bmatrix}$ *and* $C = \begin{bmatrix} .5 \\ 0 \\ .5 \end{bmatrix}$

31. Adjust the Game Matrix below with the smallest possible Integer so that all values are positive. Construct either Linear Programming Problem which can be used to find the optimal play for each of R and C. Solve the LP problem. Find the fraction of the time each player should make their choices, and find the value of the game(in regards to the original matrix form)

$$\begin{array}{cc} & C \\ R & \begin{bmatrix} 0 & -3 \\ -1 & 2 \end{bmatrix} \end{array}$$

32. Adjust the Game Matrix below with the smallest possible Integer so that all values are positive. Construct either Linear Programming Problem which can be used to find the optimal play for each of R and C. Solve the LP problem. Find the fraction of the time each player should make their choices, and find the value of the game(in regards to the original matrix form)

$$\begin{array}{cc} & C \\ R & \begin{bmatrix} -2 & 3 & 0 \\ 1 & -1 & 2 \\ 0 & 4 & 1 \end{bmatrix} \end{array}$$