Math 125 Quiz 8 Spring 2017 Name: ______________________

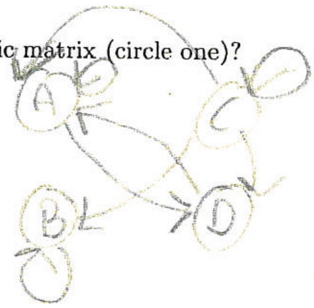
TA: Reynolds Trifunovski Zheng **Discussion Time** 8 9 10 11

1. (4 pts) A business reviews its machinery annually and does one of four things: leaves machinery in use as is, upgrades it, sells it, or destroys it. If the machine was left as is the past year, there is a 50% chance it is left as is the next year, a 25% chance it is upgraded, a 15% chance it is sold, and a 10% chance it is destroyed. If the machine was upgraded the past year, there is a 60% chance it is left as is the next year, a 0% chance it is upgraded, a 35% chance it is sold, and a 5% chance it is destroyed. If the machine is sold or destroyed, it stays that way. Give the associated absorbing matrix in STANDARD FORM.

$$\begin{array}{c|c} & \begin{array}{cccc} S & D & A & U \end{array} \\ \begin{array}{c} S \\ D \\ A \\ U \end{array} & \begin{pmatrix} 1 & 0 & .15 & .35 \\ 0 & 1 & .1 & .05 \\ 0 & 0 & .5 & .6 \\ 0 & 0 & .25 & 0 \end{pmatrix} \end{array}$$

2. (3 pts) Is the following matrix an absorbing stochastic matrix (circle one)?

$$\begin{array}{c|c} & \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{bmatrix} 0.7 & 0 & 0.1 & 0.5 \\ 0 & 1 & 0.3 & 0 \\ 0 & 0 & 0.4 & 0 \\ .3 & 0 & 0.2 & 0.5 \end{bmatrix} \end{array}$$



(a) Yes, it is.

(b) No, it is not a stochastic matrix.

(c) No, there is no absorbing state.

(d) No, one cannot get to an absorbing state from each state.

3. (4 pts) The payoff matrix from player C to player R for a game is given as:

$$\begin{bmatrix} \boxed{8} & \boxed{5} \\ 2 & -6 \\ -3 & 2 \end{bmatrix}$$

(a) Which column is player C's optimal strategy? col 2

(b) Which row is player R's optimal strategy? row 1

(c) Is this game strictly determined? Justify your answer.

yes. has a saddle point at (1,2).

(d) If this game is strictly determined, what is the value of the game? If it is not strictly determined, write "not strictly determined."

$v = 5$

4. An absorbing stochastic matrix for a Markov process in standard form is given:
$$\begin{array}{c|cccc} & A & B & C & D \\ \hline A & & & & \\ B & & & & \\ C & & & & \\ D & & & & \end{array}$$

$$\begin{array}{cc} & \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{bmatrix} 1 & 0 & 0.25 & 0.15 \\ 0 & 1 & 0 & 0.25 \\ 0 & 0 & 0.25 & 0.3 \\ 0 & 0 & 0.50 & 0.3 \end{bmatrix} \end{array}$$

(a) (3 pts) What is the fundamental matrix? Feel free to use your calculator to do computations, but indicate what matrices you are using. Identify S and R first.

$R = \begin{pmatrix} .25 & .3 \\ .5 & .3 \end{pmatrix}$

$(I-R)^{-1} = \begin{pmatrix} 1.87 & .8 \\ 1.33 & 2 \end{pmatrix}$

$S = \begin{pmatrix} .25 & .15 \\ 0 & .25 \end{pmatrix}$

(b) (3 pts) What is the stable matrix? Feel free to use your calculator to do computations, but indicate what matrices you are using. Taking the matrix to a high power is not an acceptable solution (but great for checking your work!).

$$\begin{pmatrix} 1 & 0 & 2/3 & 1/2 \\ 0 & 1 & 1/3 & 1/2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

5. (3 pts) The payoff matrix from player C to player R for a game is given as:
$$\begin{bmatrix} 2 & 5 & -4 \\ -3 & -6 & 7 \\ 4 & 4 & -3 \end{bmatrix}.$$

Suppose that R uses the mixed strategy $R = \begin{bmatrix} .1 & .5 & .4 \end{bmatrix}$ and C uses the mixed strategy $C = \begin{bmatrix} .2 \\ .3 \\ .5 \end{bmatrix}$.

Use your calculator to find the expected payoff.

$$(.1 \ .5 \ .4)\begin{pmatrix} 2 & 5 & -4 \\ -3 & -6 & 7 \\ 4 & 4 & -3 \end{pmatrix}\begin{pmatrix} .2 \\ .3 \\ .5 \end{pmatrix} = (.74)$$