

# Topics in Tensors

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<sup>1</sup>Department of Mathematics, Statistics and Computer Science, University of Illinois at Chicago, Chicago, Illinois 60607-7045, USA, e-mail: [friedlan@uic.edu](mailto:friedlan@uic.edu)






In the past ten years, tensors again became a hot topic of research in pure and applied mathematics. In applied mathematics it is driven by a multidimensional data which has a few parameters. In pure math. it algebraic geometry, i.e. secant varieties; quantum mechanics and quantum information theory; multilinear algebra. There are many interesting theoretical and numerical problems that need to be resolved. Tensors are related to matrices on one hand and on the other hand are related to polynomial maps.



To paraphrase Max Noether:

Matrices were created by God and tensors by Devil.

# The topics of 6 lectures

- 1 Ranks of 3-tensors - morn. July 6+part of afternoon?
- 2 A set theoretic solution of the salmon conjecture -  
afternoon July 6+morning July 7?
- 3 Nonnegative tensors - July 7+morn. July 8?
  - Fixed points of homogeneous monotone maps in  $\mathbb{R}_+^n$
  - Irreducibility and weak irreducibility for matrices and tensors
  - Perron-Frobenius for irreducible tensors
  - Kingman, Karlin-Ost and Friedland inequalities
  - Maxplus eigenvalues of tensors
  - Friedland-Karlin characterization of spectral radius
  - Diagonal scaling
  - Rank one approximation of tensors
  - Nonnegative multilinear forms
- 4 Approximation of tensors by low rank tensors -  
part of July 8 morning?+afternoon July 8
  - $(R_1, \dots, R_d)$ -approximation of tensors
  - Newton method on Grassmannians
  - Fast low rank approximation of matrices and tensors

-  S. Friedland, On the generic rank of 3-tensors, arXiv: 0805.3777, to appear in *Linear Algebra Appl.* .
-  S. Friedland, Positive diagonal scaling of a nonnegative tensor to one with prescribed slice sums, *Linear Algebra Appl.*, 434 (2011), 1615–1619.
-  S. Friedland, On tensors of border rank  $l$  in  $\mathbb{C}^{m \times n \times l}$ , arXiv:1003.1968, to appear in *Linear Algebra Appl.* .
-  S. Friedland, S. Gauber and L. Han, Perron-Frobenius theorem for nonnegative multilinear forms, arXiv:0905.1626, to appear in *Linear Algebra Appl.* .
-  S. Friedland and E. Gross, A proof of the set-theoretic version of the salmon conjecture, arXiv:1104.1776.

-  S. Friedland, V. Mehrmann, A. Miedlar, and M. Nkengla, Fast low rank approximations of matrices and tensors, to appear in *ELA*, [www.matheon.de/preprints/4903](http://www.matheon.de/preprints/4903).
-  S. Friedland and V. Mehrmann, Best subspace tensor approximations, arXiv:0805.4220,