Math 320 HOMEWORK ASSIGNMENTS

MATH 320 Linear Algebra I, Spring 2012
31396 - LCD - undergrad MWF 2:00-2:50, Addams Hall 306
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Homework Assignments
from the book Linear Algebra by Jim Hefferon, Mathematics Saint Michael's College. http://joshua.smcvt.edu/linearalgebra/

1 HOMEWORK ASSIGNMENT 1: 1.9 - 1.13, 2012 due 1.18.12

One.I.1 p' 9- 11: 1.16, 1.17, 1.19, 1.21 b, 1.22
One.I.2 p'18-20: 2.15-2.18, 2.20(a,b), 2.21(a,b), 2.23,

2 HOMEWORK ASSIGNMENT 2: 1.18 - 1.20, 2012 due 1.25.12

One.I.3. p29: 3.15(c), 3.16(a), 3.17,
One.III.1 p51-52: 1.7(b,d), 1.8, 1.9, 1.10, 1.11

3 HOMEWORK ASSIGNMENT 3: 1.23 - 1.27, 2012 due 2.3.12

One.III.2. p60-61: 2.10(b,e), 2.12
Two.I.1. p 86-87: 1.18(a,b,c); 1.19; 1.20d; 1.24
Two.I.2. p95-97: 2.20(a,b,c); 2.22; 2.24(a,b,c); 2.26(a,b,c).

4 HOMEWORK ASSIGNMENT 4: 1.30 - 2.3, 2012 due 2.8.12

Two.II.1. p'106-109: 1.18, 1.19, 1.29, 1.32.
Two.III.1. p'114-116: 1.16(a,b,c), 1.17, 1.20, 1.23, 1.24(a,b), 1.32(a).
Two.III.2 p' 120-121: 2.15, 2.16, 2.18(a,b), 2.19a.
Two.III.3 p' 127-128: 3.19, 3.20(a,b,d), 3.21(a,b,c), 3.24.

5 HOMEWORK ASSIGNMENT 5: 2.6 - 2.10, 2012 due 2.15.12

1. Find the rank, and a basis of the row and column spaces of the following matrices.

a. $A = \begin{bmatrix} 1 & 2 & 0 & -1 \\ 2 & 6 & -3 & -3 \\ 3 & 10 & -6 & -5 \end{bmatrix}$

b. $A = \begin{bmatrix} 0 & 0 & 3 & 1 & 4 \\ 1 & 3 & 1 & 2 & 1 \\ 3 & 9 & 4 & 5 & 2 \\ 4 & 12 & 8 & 8 & 7 \end{bmatrix}$

2. Determine which of the following matrices have the same row space.

$A = \begin{bmatrix} 1 & -2 & -1 \\ 3 & -4 & 5 \end{bmatrix}, B = \begin{bmatrix} 1 & -1 & -2 \\ 2 & 3 & -1 \end{bmatrix}, C = \begin{bmatrix} 1 & -1 & 3 \\ 2 & -1 & 10 \\ 3 & -5 & 1 \end{bmatrix}.$

Hint: Bring each matrix to RREF. Then two matrices have the same row spaces if and only if they have the same nonzero rows in their RREF.

Two.III.4 p'136: 4.20(a,d), 4.21, 4.23, 4.26, 4.34

Two.III.fields p'139: 1-5

6 HOMEWORK ASSIGNMENT 6: 2.13 - 2.17, 2012 due 2.22.12

Three.I.1 p' 164: 1.11, 1.13 (a-d)
Three.I.2 p'172: 2.8 (a-d), 2.10, 2.12.
Three.II.1 p' 179: 1.17 (a-c), 1.18 (a-c), 1.19.
Three.II.2 p' 190: 2.23 (a,c,d), 2.24 (note that the rank of the linear transformation is the dimension of its range).

7 HOMEWORK ASSIGNMENT 7: 2.17 - 2.22, 2012 due 2.27.12

Three.III.1 p'200: 1.11, 1.12, 1.14, 1.15, 1.16, 1.22, 1.26
Three.III.2 p'207: 2.9 (b,c), 2.10b, 2.13
Three.IV.1 p'212: 1.8
Three.IV.2 p'218: 2.14(a,b,c), 2.16, 2.17.
Three.IV.4 p'234: 4.14, 4.15, 4.16 (b,d,e,f).

Markov chains problems:
1. UIC plans to change their online registration system so that students can change their schedule each day, through the semester. English 999 is offered two times 10AM and 2PM. Each day, 1/10 of the students in the 10AM switch to the 2PM class, while 1/5 of the students in the 2PM switch to 10AM. The rest of the students stay in their respective classes (for that day at least).
a. Let $\mathbf{x}_n := \begin{pmatrix} AM_n \\ PM_n \end{pmatrix}$ represent the number of the students in each of these classes on day n of the semester. Give a matrix equation relation between $\mathbf{x}_{n+1}$ and $\mathbf{x}_n$.
b. 43 students are registered in English 999 on the first day of the class (this is the total for both classes.) How many students are in the 10AM section of the semester?

2. In a village of 12,000 people 1,000 people got common cold on January 1, 2000, by the end of the day. By the end of each following day, %20 percent of the healthy people got cold and %60 percent recovered by taking over the counter drugs. Let h_n and s_n be the number of healthy and sick people respectively on day n of 2000.
(a) Write down the transition matrix A such that $(h_{n+1}, s_{n+1})^\top = A(h_n, s_n)^\top$.
(b) How many sick people were by the end of January 3?
(c) How many sick people were after many days, (the steady state)?

8 HOMEWORK ASSIGNMENT 8: 2.24 - 3.5, 2012 due 3.7.12

Three.V.1 p'239: 1.6, 1.7, 1.11.
Three.V.2 p' 245: 2.10, 2.11, 2.12.
One.II.2 p' 43: 2.10, 2.11, 2.14.
Three.VI.1 p' 251: 1.7, 1.8.

9 HOMEWORK ASSIGNMENT 9: 3.07 - 3.12, 2012 due 3.14.12

Three.VI.2 p' 256–257: 2.9, 2.10, 2.11, 2.16, 2.17.
Three.VI.3 p' 264–265: 3.10, 3.11, 3.12, 3.13, 3.19, 3.20.
Three.VI.Best Fit: p' 270: 1,
Additional problem on Least Squares:
1. Find the best squares fit by a linear function to the data: $(-1, 0), (0, 1), (1, 3), (2, 9)$
2. Find the least squares solution to

$$\begin{array}{rcrcl} & x_1 & + & x_2 & = & 3 \\ - & 2x_1 & + & 3x_2 & = & 1 \\ & 2x_1 & - & x_2 & = & 2 \end{array}$$

Use this solution to show that the original system is not solvable.
3. Find the best quadratic least squares, i.e. parabola of the form $y = a + bx + cx^2$ where a, b, c are unknown, to the data $(0, 3), (1, 2), (2, 4), (3, 4)$.

10 HOMEWORK ASSIGNMENT 10 Assigned 3-13 – Due 3-28-12

Hefferon: Problem 1 from page 289.
Four.I.1 page 295: 1.1 to 1.11.
Four.I.2 page 299: 2.7, 2.8, 2.11, 2.13, 2.14.
Do the following additional problems

1. Let $\mathbf{u} = (1, -1, 1, -1)^\top, \mathbf{v} = (2, 0, -2, 1)^\top$. Find
   (a) A basis to the orthogonal complement of $\mathbf{U} := \mathrm{span}(\mathbf{u}, \mathbf{v})$.
   (b) The projection of the vector $(1, 1, 0, 0)^\top$ on $\mathbf{U}$ and $\mathbf{U}^\perp$.
2. Let $A \in \mathbb{R}^{4\times3}$. Assume that the vector $(1, -1, 1, -1)^\top$ is a vector in the column space of A. Is it possible that a vector $(2, 0, -2, 1)^\top$ is in the null space of $A^\top$? If yes give an example of such a matrix. If not, justify why.

3. Let $(-1,0),(0,1),(1,3),(2,9)$ be four points in the plane (x,y) Find the best least squares by a quadratic polynomial $y = ax^2 + bx + c$.

4. Perform the Gram-Schmidt orthonogonalization process on the vectors $\mathbf{u} = (2,0,2,-1)^\top, \mathbf{v} = (0,1,-4,1)^\top, \mathbf{w} = (4,3,1,1)^\top$

   (a) Find the QR decomposition of the matrix $A = [\mathbf{u}\ \mathbf{v}\ \mathbf{w}]$.

   (b) Complete the orthonormal basis you found uisng the Gram-Schmidt problem to an orthonormal basis of $\mathbb{R}^4$.

11 HOMEWORK ASSIGNMENT 11 Assigned 3-20 – Due 4-4-12

Hefferon:
Four.I.3 p' 308: 3.23, 3.32.
Four.II.1 p' 321: 1.8, 1.22
Four III.1 p' 327: 1.13 – 1.26.
Cramer's rule p' 330: 1a, 2, 4.

12 HOMEWORK ASSIGNMENT 12 Assigned 3-31 – Due 4-11-12

Hefferon:
Five.II.3, page 362–364: 3.20–3.30, 3.40.
Five.II.1, page 352–353: 1.4 – 1.8, 1.10–1.16.
Five.II.2, page 356–367: 2.7 – 2.18.

13 HOMEWORK ASSIGNMENT 13 Assigned 4-11 – Due 4-18-12

A. Let $A = \begin{bmatrix} 2 & -1 \\ -2 & 3 \end{bmatrix}$.

1. Find eigenvalues and corresponding eigenvectors.
2. Find a nonsingular X and a diagonal matrix D such that $A = XDX^{-1}$.
3. Find A^4 and $f(A)$ where $f(t) = t^4 - 5t^3 + 7t^2 - 2t + 5$.
4. Let $B = \frac{1}{4}A$. Find $\lim_{k\to\infty} B^k$.
5. Find C such that $C^2 = A$.

B. Find the general and the particular solution of the differential systems, if asked.

1. $\begin{array}{rl} y_1' = & 3y_1 + 4y_2 \\ y_2' = & 3y_1 + 2y_2 \end{array}, \quad y_1(0) = 6,\ y_2(0) = 1.$

2. $\begin{array}{rl} y_1' = & -y_1 + 2y_2 \\ y_2' = & 2y_1 - y_2 \end{array}, \quad y_1(0) = 3,\ y_2(0) = 1.$

3. $\begin{array}{rl} y_1' = & y_1 - y_2 \\ y_2' = & y_1 + y_2 \end{array}.$

C. Let A be a diagonable matrix whose eigenvalues are 1 or -1. Show that A is invertible and $A^{-1} = A$.
D. Recall that a square matrix A is defective if it has an eigenvalue λ of mulitplicity $m \geq 2$ but the dimension of the null space of $A - \lambda I$ is at most $m - 1$.

1. Show that the matrix $\begin{bmatrix} a & 1 & 0 \\ 0 & a & 1 \\ 0 & 0 & b \end{bmatrix}$ is defective for all values of a abd b.

2. Find the value of α for which the below matrix is defective or show that such value does not exists $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & \alpha \end{bmatrix}$.

14 HOMEWORK ASSIGNMENT 14 Assigned 4-17 – Due 4-25-12

A. Find an orthogonal matrix Q and a diagonal matrix D such that $A = QDQ^\top$ for the following cases:

1. $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$,

2. $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & -2 \\ 1 & -2 & 3 \end{bmatrix}$,

3. $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$,

4. $\begin{bmatrix} 4 & 2 & -2 \\ 2 & 1 & -1 \\ -2 & -1 & 1 \end{bmatrix}$.

B. Let $A = [a_{ij}]_{i=j=1}^n$ be a real symmetic matrix. Let $\lambda_1(A)$ and $\lambda_n(A)$ be the maximum and the minimum eigenvalues of A respectively. Let α and β be the maximum and the minimum of the diagonal entries $a_{11}, \ldots, a_{nn}$.

1. Use Rayleigh quotient, page 234 of [1], to show that $\lambda_1(A) \geq \alpha, \beta \geq \lambda_n(A)$. Use the results of Problem A, to see how good these inequalities are.

2. Let M be the maximum of $|a_{ij}|, i, j = 1, \ldots,$. Show that $|\mathbf{x}^\top A\mathbf{x}| \leq M \sum_{i=j=1} |x_i||x_j| = M(\sum_{i=1}^n |x_i|)^2$,where $\mathbf{x} = (x_1, \ldots, x_n)^\top$. Use Cauchy-Schwarz inequality to show that $|\mathbf{x}^\top A\mathbf{x}| \leq Mn\|\mathbf{x}\|^2$, where $\|\mathbf{x}\| = \sqrt{\sum_{i=1}^n x_i^2}$. Deduce that $\lambda_1(A) \leq Mn$ and $\lambda_n(A) \geq -Mn$.

3. Show that if $a_{ij} = 1$ for all i, j then $\lambda_1(A) = n$.

C. For the following matrices square matrices A find LU decomposition, where L is a lower triangular with 1 on the diagonal, and U is upper triangular. If A is symmetric then find $LDL^\top$ decomposition. See pages 237–245 in [1]. For a symmetric matrix A use $LDL^\top$ decomposition to determine how many positive and negative eigenvalues A has, without computing the eigenvalues. (The number of positive and negative eigenvalues of a symmetric matrix of the form $A = LDL^\top$ is equal to the number of positive and negative diagonal entries of D.)

1. $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & 1 \\ -3 & 1 & -2 \end{bmatrix}$,

2. $\begin{bmatrix} 1 & -1 & 2 \\ -1 & 3 & -2 \\ 2 & -2 & 5 \end{bmatrix}$,

3. $\begin{bmatrix} 1 & -1 & 2 \\ -1 & 3 & -2 \\ 2 & -2 & 3 \end{bmatrix}$,

References

[1] S. Friedland, Outline of Lectures in Linear Algebra Math 320, http://homepages.math.uic.edu/∼friedlan/math320lecS12.pdf

[2] J. Hefferon, *Linear Algebra*, http://joshua.smcvt.edu/linearalgebra/Linear Algebra