Math 320, Linear Algebra, Practice Problems for Final Math 320, May 1, 2013, Instructor: S. Friedland

The final will cover §3.14 to §6.13 in my notes, excluding §6.10 and §6.11.
Linear Algebra I - Lectures Notes - Spring 2013
http://homepages.math.uic.edu/~friedlan/math320lecS13t.pdf
§3.14-§6.5 are also covered in Hefferon book: http://joshua.smcvt.edu/linearalgebra/, pages 229-331, 353-372.

Name and University e-mail address here and in your exam booklet:

Show all work. **Unjustified** answer yields no credit.

I. Let $A \in \mathbb{C}^{n\times n}$

1. Define an eigenvalue and eigenvector of A.
2. Show that A is singular if and only if A has a zero eigenvalue.
3. Assume that A has m different eigenvalues $\lambda_1, \ldots, \lambda_m$ with corresponding eigenvectors $\mathbf{x}_1, \ldots, \mathbf{x}_m$. Show that $\mathbf{x}_1, \ldots, \mathbf{x}_m$ are linearly independent.

II. Let $A = \begin{bmatrix} 0.9 & 0.3 \\ 0.1 & 0.7 \end{bmatrix}$.

1. Find the eigenvalues and the corresponding eigenvectors.
2. Find an invertible X and a diagonal D such that $A = XDX^{-1}$.
3. Find the limit of A^m when $m \to \infty$.

III. Consider the system of differential equations:

$$\begin{aligned} y_1' &= -7y_1 + 18y_2 \\ y_2' &= -3y_1 + 8y_2 \end{aligned}$$

1. Find the general solution of the system.
2. Find the solution satisfying the initial condition $y_1(0) = 1, y_2(0) = -1$.

IV. Let $A = \begin{bmatrix} 0 & -1 & -2 \\ -1 & 0 & 2 \\ -2 & 2 & 3 \end{bmatrix}$. Assume that $\det(A - \lambda I) = (5 - \lambda)(\lambda + 1)^2$.

1. Find a basis for the eigenspace for each eigenvalue of A.
2. Use Gram-Schmidt process to find an orthonormal basis for each eigenspace of A.
3. Find the projection of $(1, 1, 1)^T$ the eigenspace corresponding to the eigenvalue $\lambda = -1$.
4. Find an orthogonal matrix Q and a diagonal matrix D, such that $A = QDQ^T$.

V. Let $A = \begin{bmatrix} 0 & 0 & 0 \\ 2 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$.

1. Find the eigenvalues of A and the corresponding eigenvectors.
2. Is A diagonable?
3. Write down the power series for e^B for any square matrix B.
4. Use the above power series to compute e^A for 3×3 matrix given in this problem.

VI.

1. Let $A, B \in \mathbb{C}^{n\times n}$. What does it mean A and B are similar?
2. Show that if A and B are similar then A and B have the same characteristic polynomial.
3. Show that if A and B similar then A and B have the same trace and determinant.
4. Give an example of two 2×2 matrices which have the same characteristic polynomial by not similar.
5. Let $A = \begin{bmatrix} 0 & -1 & -2 \\ -1 & 0 & 2 \\ -2 & 2 & 3 \end{bmatrix}, B = \begin{bmatrix} 0 & -1 & -2 \\ -1 & 0 & 2 \\ -2 & 2 & 2 \end{bmatrix}$. Show that A is not similar to B.

VII.

1. Assume that $A, B, C \in \mathbb{R}^{n\times n}$ are invertible. Write done the expressions for $(ABC)^\top, (A^\top)^{-1}, (ABC)^{-1}$.
2. Suppose that $A \in \mathbb{R}^{n\times n}$ is a symmetric invertible matrix. Is A^{-1} symmetric? If yes, give a short proof, if not give a simple counterexample.
3. Let $A, B \in \mathbb{C}^{n\times n}$ and assume that AB is singular. Show that either A or B are singular.
4. Let $A = \begin{bmatrix} 1 & -1 & -2 \\ -1 & 0 & 2 \\ 0 & 2 & -1 \end{bmatrix}$. Find the determinant and the inverse of A.

VIII.

1. Find the projection of $(1, 1, 1, 1)^\top$ on the subspace spanned by $(1, 1, -1, 1)^\top$ and $(1, 0, -1, 1)^\top$ using least squares.
2. Fit the best line through the points $(-1, 1), (0, 2), (1, -1)$.
3. Let $\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3 \in \mathbb{R}^4$ be 3-linearly independent vectors. Let $\mathbf{U}$ be the subspace spanned by these three vectors. What is the dimension of $\mathbf{U}^\perp$?