MCS 361, Discrete Mathematics, **Practise Test 2**,
April 3, 2009, Instructor: S. Friedland
Name and e-mail address:

No books or notes. Show all your work. Write solutions in the exam booklet without copying the problems. **Unjustified** answer yields no credit.

Problem 1. Let G be simple undirected graph, (no loops), on n vertices with e edges, and the degree sequence $d_1, \ldots, d_n$. (So the vertices are labeled $1, \ldots, n$ and the degree of the vertex i is d_i.)
(a) Can e be an odd number? (if yes give an example, if no give a short argument.)

(b) Can $d_1 + d_2 + \ldots + d_n$ be an odd number. (if yes give an example, if no give a short argument.)

(c) Let G be the graph on 4 vertices labeled $1, 2, 3, 4$. Let E be the following set of edges:

$$E = \{\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{3,4\}\}.$$

Write down the adjacency matrix of this graph.

Problem 2. Consider the graph given in Figure 1 at the end of the exam.

(a) Does it have a Eulerian cycle or path? If yes, write down one. If no, give explain why.

(b) Does this graph has a Hamiltonian cycle or path? If yes, write down one. If no, give explain why.

(c) What is the chromatic number of this graph.

Problem 3. Consider the weighted graph given in Figure 2 at the end of the exam. Find a minimum spanning tree.

Problem 4.
a. In a Chinese restaurant you have a choice of 3 soups, 5 appetizers, 10 main dishes and 2 deserts. You have a deal: for the price of the main dish you can choose in addition two extra different dishes. (You are not allowed to have two soups, two appetizers, two main dishes, two deserts.) How many choices you have for these three dishes?

b. How many ways can a committee of n members form two disjoint, nonempty subcommittees if not everybody has to serve on either committee.

Problem 5. You have 10 runners.

a. You choose a team of q runners with $q = 3$, $q = 5$, $q = 7$, or $q = 9$. How many choices are there?

b. You choose 2 runners to run marathon, 5 runners to run $10K$ and 3 runners to run 800 meters. How may choices are there?