MCS 361, Discrete Mathematics, **Practise Final Test**,
April 29, 2009, Instructor: S. Friedland
Name and e-mail address:

No books or notes. Show all your work. Write solutions in the exam booklet without copying the problems. **Unjustified** answer yields no credit.

Problem 1. There are five persons A, B, C, D, E and 5 jobs $1, 2, 3, 4, 5$. The performance of each person at each job is given by the following table.

	1	2	3	4	5
A	0.2	0.4	0.3	0.5	0.6
B	0.3	0.2	0.6	0.6	0.7
C	0.5	0.3	0.4	0.4	0.7
D	0.7	0.4	0.4	0.3	0.5
E	0.6	0.5	0.3	0.4	0.7

a. (15 pts) Find an optimal assignment of 5 persons to perform all 5 jobs, by using the greedy algorithm for the following order: E,A,B,C,D.
b. (5 pts) Is your assignment best possible? **Justify!**

Problem 2. Consider this algorithm:

Algorithm SumMystery
procedure SqSum(**in** N; **out** k, S)
$k \leftarrow 0$; $S \leftarrow 0$
repeat until $S > N$
$k \leftarrow k + 1$
$S \leftarrow S + k^2$
endrepeat
endpro

1. (6 pts) What values of k and S does SqSum(3; k, S) return?
2. (8pts) What values of k and S does SqSum(14; k, S) return?
3. (6pts) Describe briefly what value of S does SqSum(N; k, S) return?

Problem 3. (20 pts.) Consider the sequence $a_1, a_2, \ldots$, where $a_n = a_{n-1} + 2a_{n-2}$ for $n \geq 3$. Show
(a) if a_1 and a_2 integers then each a_n is an integer.
(b) Assume that $a_1 = 1, a_2 = 2$. Guess the formula for a_n and prove this formula by induction.

Problem 4. (20pts)

1. Draw a complete graph on 5 vertices. Is it planar?
2. Draw a two colorable graph on 5 vertices with the maximal number of edges. Is it planar?
3. Problem 1 can be stated in terms of a graph on 10 vertices with weighted edges. Is this graph is a complete graph, a bipartite graph, or neither of the two above. **Explain**

Problem 5. (20pts) Consider the graph given in Practice Test 2 *figures* on my website.
(a) Does it have a Eulerian cycle or path? If yes, write down one. If no, give explain why.
(b) Does this graph has a Hamiltonian cycle or path? If yes, write down one. If no, give explain why.
(c) What is the chromatic number of this graph.

Problem 6. (20 pts) There are five towns numbered $1, 2, 3, 4, 5$ in some country. The cost of digging cable connection between any two towns are given by the following table.

	1	2	3	4	5
1		150	175	125	200
2			225	250	160
3				210	120
4					155

1. This problem corresponds to a graph with weighted edges. How many vertices and how many edges this graphs has. How de we call such a graph?
2. Find the minimal number of cable connections with the minimal costs which will connect all the cities.

Problem 7. (20pts)
a. In a Chinese restaurant you have a choice of 3 soups, 5 appetizers, 10 main dishes and 2 deserts. You have a deal: for the price of the main dish you can choose in addition two extra different dishes. (You are not allowed to have two soups, two appetizers, two main dishes, two deserts.) How many choices you have for these three dishes?
b. How many ways can a committee of n members form two disjoint, nonempty subcommittees if not everybody has to serve on either committee.

Problem 8. (20 pts)

1. In how many ways you can put 27 marked balls into 4 marked urns. (Do not compute the exact number.)
2. In how many ways you can put 27 unmarked balls into 4 marked urns, such that each urn contains at least two balls. (Compute the exact number A.)
3. Is the number of ways you can put 27 unmarked balls in 4 unmarked urns, such that at least each urn contains at least two balls, is greater or less than $\frac{A}{4!}$? Explain.

Problem 9. (20 pts.) You have 3 identical boxes. One contains 2 one dollar bills. One contains 2 five dollar bills. One contains 1 one dollar bill and 1 five dollar bill. You choose at random, with the same same probability, one box, and then withdraw a bill. You look at the bill and it is a five dollar bill. What are the chances that the second bill is also a five dollar bill?

Problem 10. (20 pts.)

1. A fair die is tossed twice. The random variable is the sum of the two tosses. Find the two results for which the probabilities of the outcome are the highest and the lowest, and give their probabilities.
2. You observe a magician tossing the same coin 450 times. Out of that 150 times the Heads showed up. Assume that the probability of having Head is constant and equal to p. Suppose furthermore that the tosses are independent.
   (a) Estimate p.
   (b) Estimate the probability that out of 288 tosses the number of heads will be between 80 and 112?