Problem 1. Consider the system
\[
\begin{align*}
  x_1 + 2x_2 - 3x_3 + x_4 &= 1 \\
  -x_1 - x_2 + 4x_3 - x_4 &= 6 \\
  2x_1 - 4x_2 + 6x_3 - x_4 &= 1
\end{align*}
\]
a. Write down the augmented matrix \( B \) associated with this system.
b. Apply the Gauss-Jordan reduction to bring the augmented matrix into the reduced row form.
c. List all lead and all free variables. Find the set of solutions to the system.

Problem 2. Consider the system
\[
\begin{align*}
  x_1 + 2x_2 + 3x_3 &= 0 \\
  2x_1 + 5x_2 + 4x_3 &= 0 \\
  x_1 + 3x_2 + ax_3 &= 0
\end{align*}
\]
For which values of \( a \) this system:
a. Is solvable;
b. Has infinite number of solutions.
**Hint:** Bring this system to a Row Echelon Form.

Problem 3. True or False. (Justify briefly!):
a. The set \( S := \{ x = (x_1, x_2)^\top \in \mathbb{R}^2, \ 3x_1 + x_2^2 = 0 \} \) is a subspace of \( \mathbb{R}^2 \).
b. Any 3 vectors in \( \mathbb{R}^4 \) can not form a basis in \( \mathbb{R}^4 \).

Problem 4. Consider the transformation \( L : \mathcal{P}_2 \to \mathcal{P}_2, \ L(p(x)) = -p'(x) + p(1)x^2. \) (\( \mathcal{P}_2 \) is the vector space of all polynomials of degree 2 at most, and \( p' \) is the derivative of \( p \).)
a. Show that \( L \) is a linear transformation.
b. Find the representation matrix of \( L \) in the standard basis \( \{1, x, x^2\} \) of \( \mathcal{P}_2 \).
c. Is \( L \) isomorphism? **Justify!**

Problem 5. Let \( \mathbb{R}^{2\times2} \) be the set of all \( 2 \times 2 \) matrices \( A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \). Recall that \( A^\top = \begin{bmatrix} a & c \\ b & d \end{bmatrix} \).
a. Let \( S_2 \subset \mathbb{R}^{2\times2} \) be the sets of symmetric matrices, i.e. all \( 2 \times 2 \) matrices satisfying \( A^\top = A \). Show that \( S_2 \) is a subspace of \( \mathbb{R}^{2\times2} \) and find its dimension. (**Hint:** Find the general form of \( A \) in \( S_2 \)).
b. Let \( AS_2 \subset \mathbb{R}^{2\times2} \) be the set of all skew symmetric matrices, i.e. all \( 2 \times 2 \) matrices satisfying \( A^\top = -A \). Assume that \( AS_2 \) is a subspace of \( \mathbb{R}^{2\times2} \). Find its dimension. (**Hint:** Find the general form of \( A \) in \( AS_2 \)).
c. Is \( \mathbb{R}^{2\times2} = S_2 \oplus AS_2 \)? Justify briefly.