MCS 421 -Combinatorics, **Practise Test**,
February 22, 2013, Instructor: S. Friedland
Name and e-mail address:

No books or notes. Show all your work. Write solutions in the exam booklet without copying the problems. You can use a result (x) of any part of the problem, to show other part of any problem. **Unjustified** answer yields no credit.

Problem 1.
a. (10p.) In how may ways six indistinguishable rooks be placed on a 6-by-6 board so that not two rooks can attack one another?
b. (10 p.) In how many ways if there are three reds, two blue and one black?
Problem 2. Given a multiset $S = \{3 \cdot a, 4 \cdot b, 5 \cdot c\}$
a. (10pts) Determine the number of 11-permutations of S.
b. (10pts) Determine the number of all combinations (of any size) of S.
Problem 3.
a. (15 pts.) In a room there are 10 people, none of whom are older the 60, (ages are given in whole numbers only), but each of whom is at least one year old. Show that we can always find two groups of people (with no common person) the sum of whose ages is the same.
b. (5 pts) show that this is not true if the number of people is 5.
Problem 4.
a. (10 points.) Show that there exists 6 integers $a_1, \ldots, a_6$ such that for each $1 \le i < j \le 6$ the integer $a_j - a_i$ is not divisible by 6
b. (10 points.) Show that for any 7 integers $a_1, \ldots, a_7$ there exists $1 \le i < j \le 7$ such that $a_j - a_i$ is divisible by 6.
Problem 5.
a. (6 pts) What is the maximal number of inversions for a permutation on $\{1, 2, 3, 4, 5\}$? Right down all these permutations.
b. (14 pts.) On the set of positive integers $\mathbb{N}$ define the following relation R. aRb if $a = 2^k b$ for some integer k. Show that this is an equivalence relation. Describe each equivalence class.
Problem 6.
a. (10p.) Consider the three-dimensional grid whose dimensions are 10 by 15 by 20. You are at the front lower left corner of the grid and wish to get to the back upper right corner 45 “blocks” away. How many different routes are there in which you walk exactly 45 blocks.
b. (10 p.) Determine the coefficient of $x_1^3 x_2 x_3^4 x_5^2$ in the expansion $(x_1 + x_2 + x_3 + x_4 + x_5)^{10}$.
Problem 7.
a. (10pts) Determine a general formula of the number of permutations of the set $\{1, \ldots, n\}$, where $n \ge 5$, such that exactly 3 integers are in their natural place.
b. (10pts) At a party, seven gentlemen check their hats. In how many ways can their hats be returned so that at least two of the gentlemen receive their own hats?