Linear Algebra II
Practise Quiz 1

MATH 425 Linear Algebra II, Spring 2012
LCD-undergrad 24908; LCD-grad 24909,
MWF 10:00-10:50, Addams Hall 303

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1 Practise problems

1. Explain why $\mathbb{Z}_7$ is a field and $\mathbb{Z}_8$ is not.
2. Suppose that a vector space $\mathbf{V}$ is spanned by 3 vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$.
    1. Show that if $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3 \in \mathbf{V}$ are linearly independent then any vector $\mathbf{v} \in \mathbf{V}$ is a unique linear combination of $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3$.
    2. Are $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ linearly independent? **Justify!**
3. Denote by $\mathcal{P}_n$ the set of all polynomials of degree n at most with real coefficients.
    1. Show that this is a vector space over reals. What is the zero element of this space.
    2. Show that the polynomials $1, t, t^2, \ldots, t^n$ are linearly independent.
    3. What is the dimension of $\mathcal{P}_n$?
    4. Fix $a \in \mathbb{R}$. Consider all polynomials in $\mathcal{P}_n$ satisfying $p(a) = b$. For which values of b it is a subspace of $\mathcal{P}_n$? What is the dimension of this subspace? For these values find a basis in this subspace.

References

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[3] G.H. Golub and C.F. Van Loan. Matrix Computation, *John Hopkins Univ. Press, 3rd Ed.*, Baltimore, 1996.

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