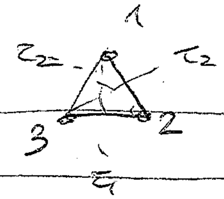
MCS-421 – HW11 – Nov. 29, 2010 – S. Friedland

19.



id – $[1]\circ[2]\circ[3]$ – z_1^3

$\rho = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$ $[1\,2\,3]$ – z_3

$\rho^2 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$ $[1\,3\,2]$ z_3

$\tau_1 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$ $[1]\circ[2\,3]$ $z_1 z_2$

$\tau_2 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$ $[1\,3]\circ[2]$ $z_1 z_2$

$\tau_3 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$ $[1\,2]\circ[3]$ $= z_1 z_2$

$P_{D_3} = \frac{1}{6}(z_1^3 + 3z_1z_2 + 2z_3) = P_{D_3}(z_1, z_2, z_3)$

$P_3(2,2,2) = \frac{1}{6}(8 + 3\cdot 4 + 2\cdot 2) = \frac{1}{6}(8+12+4) = \frac{24}{6} = 4$

$P_3(p,p,p) = \frac{1}{6}(p^3 + 3p^2 + 2p) = \frac{p(p^2+3p+2)}{6} = \frac{p(p+1)(p+2)}{6}$

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26. 4R, 3B – 7 beads. So we need to ~~study~~ find $P_{D_7}(z_1, \dots, z_7)$.

id – z_1^7; ρ $[1234567] \to z_7$. Since 7 is prime

$\rho^i \to z_7$ for $i = 1, \dots, 6$. Transposition τ fix one elements and switch elements of 3 pairs e.g. $[1]\circ[2\,7]\circ[3\,6]\,[4\,5] \to z_1 z_2^3$

$P_{D_7} = \frac{1}{14}(z_1^7 + 6z_7 + 7z_1z_2^3)$

$P_{D_7}(r,b) = \frac{1}{14}\left[(r+b)^7 + 6(r^7+b^7) + 7(r+b)(r^2+b^2)^3\right]$

coefficient of r^4b^3 $\nearrow$ $\binom{7}{4} = \binom{7}{3} = \frac{7\cdot 6\cdot 5}{2\cdot 3} = 35$

$7(r+b)(r^6 + 3r^4b^2 + 3r^2b^4 + b^6)$ – coefficient of r^4b^3

$= 7(3r^4b^3) = 21$,

So $\frac{35+21}{14} = \frac{56}{14} = \underline{\underline{4}}$

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33. Suppose that the cycle decomposition of π is $[i_1 i_2 \dots i_{e_1}]\,[i_{e_1+1}, \dots, i_{e_1+e_2}]$
$[i_{e_1+\dots+e_{k-1}+1} \dots i_n]$

Note that the inverse of the cyclic permutation $[i_1, \dots, i_{e_1}]$ is $[i_1 i_{e_1} i_{e_1-1} \dots i_2]$

Hence π^{-1} has the same type as π.

35. D_6 id – z_1^6, $\rho - z_6$, ρ^2: [1 3 5]·[2 4 6] – z_3^2

ρ^3: [1 4] [2 5] [3 6] z_2^3 $\rho^4 \equiv \rho^2 - z_3^2$, $\rho^5 \equiv \rho$ z_6

τ = [1] [2 6] [3,5] [4] $z_1^2 z_2^2$ – 6 all together

$$P_{D_6} = \frac{1}{12}\left[z_1^6 + 2z_6 + 2z_3^2 + z_2^3 + 6 z_1^2 z_2^2\right]$$

$$P_{D_6}(k,\dots,k) = \frac{1}{12}\left[k^6+2k+2k^2+k^3+6k^4\right]$$

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45. p odd prime ρ^j has one cycle for $j=1,\dots,n-1$

Need to show that $[\rho^j(1)]^q \neq 1$ iff $q=n$.

$(\rho^j)^q(1) = \rho^{jq}(1) = jq \equiv 1 \pmod n$.

Suppose that $1 \le q \le n-1$

We need to show that $j \to qj$ maps

$\{1,\dots,n-1\}$ on $\{1,\dots,n-1\}$.

Indeed if $j_1 q = j_2 q \pmod n \Rightarrow n \mid (j_2 - j_1) q$

since $1\le q < n-1$ and a prime $n \mid (j_2-j_1)$

so for $j_1, j_2 \in \{1,\dots,n-1\}$ $1 \le j_2 - j_1 \le n-1$

not divisible by n. QED.

46. D_n → id → z_1^n, ρ^j $j=1,\dots,n-1$ z_n

τ → one fixed all other cycles of order 2: $z_1 z_2^{\frac{n-1}{2}}$

number n.

$$P_{D_n} = \frac{1}{2n}\left(z_1^n + (n-1)z_n + n z_1 z_2^{\frac{n-1}{2}}\right)$$

$$P_{D_n}(k,\dots,k) = \frac{1}{2n}\left(k^n + (n-1)k + n k^{\frac{n+1}{2}}\right)$$

48. G:

1	2	3
4	5	6
7	8	9

id – z_1^9 ρ = [1 3 9 7] [2 6 8 4] [5] $z_1 z_4^2$

ρ^2 = [19] [28] [37] [64] [5] – $z_1 z_2^4$

$\rho^3 = \rho$ $z_1 z_4^2$, τ = [13] [46] [79] [2] [5] [8]

$$P_G = \frac{1}{8}\left[z_1^9 + 2z_1z_4^2 + z_1 z_2^4 + 4 z_2^3 z_1^3\right]$$ – 4 such

$$P_G(2,2) = \frac{1}{8}\left[2^9+2^4+2^5+4\cdot 2^6\right] = 2^6+2+2^2+2^5 = 64+2+4+32 = 102$$