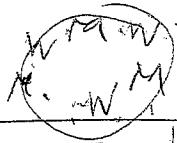


(8)



Place first M1
Then start to count
Then other men M2.. M6

(2)

have 5! choices. Now place
W1 W6 in 6 remaining chairs $\rightarrow 6!$
Total $5! \cdot 6!$

(11)

you need first to consider all possible sets of 3 integer from $[20] = \{1, \dots, 20\}$

Since it is a set they must be distinct. Hence

$$\text{the number of such sets } \binom{20}{3} = \frac{20 \cdot 19 \cdot 18}{1 \cdot 2 \cdot 3} = \frac{10 \times 19 \times 18}{3}$$

Now let us count all sets which have 2 consecutive integers.

I. Count the pairs of consecutive integers

$\langle 1, 2 \rangle, \langle 2, 3 \rangle, \dots, \langle 19, 20 \rangle$ # of pairs 19

To each pair $\langle i, i+1 \rangle$ let us take

j in $[20]$ but not different from i and $i+1$

of j is 18. So in this way we get $19 \cdot 18$

triples of 3 distinct members with two consecutive integers. However the triple consecutive numbers

$\langle 0, 1, 2 \rangle$ were counted twice $\langle i, i+1 \rangle \cup \langle i+1, i+2 \rangle$

and $\langle i+1, i+2 \rangle \cup \langle i, i+1 \rangle$.

The number of triples is $\langle 1, 2, 3 \rangle, \langle 2, 3, 4 \rangle, \dots, \langle 18, 19, 20 \rangle$

is 18. Hence the number of 3 integers

with two consecutive integers is $19 \cdot 18 - 18 = 18 \cdot (19 - 1) = 18 \cdot 18^2$

So the number we look for is $\binom{20}{3} - 18^2 =$

$$18 \times \left(\frac{10 \times 19}{3} - 18 \right) = 18 \times (190 - 54) = 6 \times 136 = 816$$

16. Each r subset of n elements is determined by its complement, i.e. the elements which are not there.

(284)

Take 8 objects R and 8 objects D and put them

RR DDD RRRR DDDDD RR That describes the path

(2R then 3D then 4R then 5D then 2R) so the number $\binom{16}{8} = \frac{16!}{8! \cdot 8!}$

MCS-421 - Combinatorics, FALL 2010
 SOLUTIONS TO HW PROBLEMS

(1)

9.2.10: HW1: CHAPTER 2, R. BRUALDI - 1, 2, 5(a, b), 8, 11, 16, 28a

1. (a) - $P(5, 4) = 5 \cdot 4 \cdot 3 \cdot 2 = 120$

(a) & (b) Last number ^(digit) is either 2 or 4 - 2 choices.

Third digit $5-1=4$ choices, Second digit, 3 choices,

First digit 2 choices: $2 \cdot 4 \cdot 3 \cdot 2 = 48$

~~1~~ $5 - 5 \cdot 5 \cdot 5 = 5^4$

(b) Last digit 2 or 4, all other digits 1-5.
 $2 \cdot 5^3$

2. Every ~~card in a suit~~ ^{suit} can be arranged in $13!$ since there are 13 cards in a suit. So all possible rearrangements of 4 suits is $(13!)^4$.

Now there are $4!$ permutations of 4 suits.

So the answer is $4! (13!)^4$

5. (a) The number of even in 50 is $50/2 = 25$

So this gives 2^{25} of factor in 50!. The number of numbers divisible by 4 is $50/4 = 12.5 \Rightarrow$ is 12 so you have extra 2^{12}

divisible by 8 is $\lfloor 50/8 \rfloor = 6$ so you have extra 2^6

" " 16 is $\lfloor 50/16 \rfloor = 2$ " " " 2^2

" " 32 is $\lfloor 50/32 \rfloor = 1$ " " " 2^1

all together 50! has exactly in his decomposition $2^{25+12+6+1} = 2^{44}$

The number of number from 1 to 50 divisible by 5 is $\lfloor \frac{50}{5} \rfloor = 10$

by 5^2 $\lfloor \frac{50}{25} \rfloor = 2$
 So 50! has $5^{10+2} = 5^{12}$ in his decomposition

Hence 50! is exactly divisible by $2^{44} \cdot 5^{12} = 10^{12}$

(b) $(1000)! = 2^a \cdot 5^b \cdot 3^c$

Clearly $b < a$ so we need to know only b .

$\lfloor \frac{1000}{5} \rfloor = 200$ $\lfloor \frac{1000}{25} \rfloor = 40$ $\lfloor \frac{1000}{125} \rfloor = 8$ $\lfloor \frac{1000}{625} \rfloor = 1$

So $b = 200 + 40 + 8 + 1 = 249$

So $(1000)!$ has exactly $\boxed{249}$ ^{last} zeros
 divisible exactly by 10^{249}