(8) (2)

W M W
M W M

Place first M1. Then start to count.
Then other men M2 .. M6
have 5! choices. Now place
W1 ... W6 in 6 remaining chairs $\to 6!$
Total $5!\,6!$

(11) You need first to consider all possible sets of 3 integer from $[20] = \{1, \ldots, 20\}$
Since it is a set they must be distinct. Hence the number of such sets $\binom{20}{3} = \frac{20 \cdot 19 \cdot 18}{1 \cdot 2 \cdot 3} = \frac{10 \times 19 \times 18}{3}$.

Now let us count all sets which have 2 consecutive integers.

I. Count the pairs of consecutive integers
$\{1,2\}, \{2,3\}, \ldots, \{19,20\}$ # of pairs 19
To each pair $\{i, i+1\}$ let us take j in $[20]$ but ~~not~~ different from i and $i+1$
of j is 18. So in this way we get $19 \cdot 18$ triples of 3 distinct numbers with two consecutive integers. However the triple consecutive numbers $\{i, i+1, i+2\}$ were counted twice $\{i, i+1\} \cup \{i+2\}$ and $\{i+1, i+2\} \cup \{i\}$.
The number of triples is $\{1,2,3\}, \{2,3,4\}, \ldots \{18,19,20\}$ is 18. Hence the number of 3 integers with two consecutive integers is $19 \times 18 - 18 = 18 \times (19-1) = 18^2$
So the number we look for is $\binom{20}{3} - 18^2 =$
$18 \times \left(\frac{10 \times 19}{3} - 18\right) = 18 \times \left(\frac{190 - 54}{3}\right) = 6 \times 136 = \underline{\underline{816}}$

16. Each r subset of n elements is determined by its complement, i.e. the elements which are not there.

(28a) Take 8 objects R and 8 objects D and put them
RR DDD RRRR DDDDD RR That describes the path
(2R then 3D then 4R then 5D then 2R) so the number $\binom{16}{8} = \frac{16!}{8!\,8!}$

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SOLUTIONS TO HW PROBLEMS

9.2.10: HW1. CHAPTER 2, R. BRUALDI - 1, 2, 5(a, b), 8, 11, 16, 28a

1. (a) - $P(5,4) = 5\cdot4\cdot3\cdot2 = 120$

(a) & (b) Last number (digit) is either 2 or 4 - 2 choices.
Third digit $5-1=4$ choices, Second digit, 3 choices,
First digit 2 choices: $2\cdot4\cdot3\cdot2 = 48$

(a) - $5\cdot5\cdot5\cdot5 = 5^4$

(b) Last digit 2 or 4, all other digits 1-5
$2\cdot5^3$

2. Every suit ~~card in a suit~~ can be arranged in $13!$
since there are 13 cards in a suit. So all possible
rearrangements of 4 suits is $(13!)^4$.
Now there $4!$ permutation of 4 suits.
So the answer is $4!\,(13!)^4$

5. (a) The number of even in 50 is $50/2 = 25$
So this gives 2^{25} of factor in $50!$. the number of numbers
divisible by 4 is $50/4 = 12.5 \Rightarrow$ is 12 so you have extra 2^{12}
divisible by 8 is $\lfloor 50/8 \rfloor = 6$ so you have extra 2^6
" " 16 is $\lfloor 50/16 \rfloor = 2$ " " " 2^2
" " 32 is $\lfloor 50/32 \rfloor = 1$ " " 2^1
all together $50!$ has exactly in his decomposition
$2^{25+12+6+1} = 2^{44}$

The number of number from 1 to 50 divisible by 5 is $\lfloor \frac{50}{5} \rfloor = 10$
" " " " " by 5^2: $\lfloor \frac{50}{25} \rfloor = 2$
So $50!$ has $5^{10+2} = 5^{12}$ in his decomposition
Hence $50!$ is exactly divisible by $2^{12}5^{12} = 10^{\boxed{12}}$

(b) $(1000)! = 2^a 5^b 3^c \ldots$

Clearly $b < a$. So we need to know only b.

$\left\lfloor \frac{1000}{5} \right\rfloor = 200 \quad \left\lfloor \frac{1000}{25} \right\rfloor = 40 \quad \left\lfloor \frac{1000}{125} \right\rfloor = 8 \quad \left\lfloor \frac{1000}{625} \right\rfloor = 1$

so $b = 200 + 40 + 8 + 1 = 249$.

So $(1000)!$ has exactly $\boxed{249}$ last zeros
divisible exactly by 10^{249}