2.10be

(b) The first is this.

$$\xrightarrow[-5\rho_1+\rho_3]{-3\rho_1+\rho_2}\begin{pmatrix}1&0&2\\0&-1&-5\\0&-1&-5\end{pmatrix}\xrightarrow{-\rho_2+\rho_3}\begin{pmatrix}1&0&2\\0&-1&-5\\0&0&0\end{pmatrix}\xrightarrow{-\rho_2}\begin{pmatrix}1&0&2\\0&1&5\\0&0&0\end{pmatrix}$$

The second is this.

$$\xrightarrow{-2\rho_1+\rho_3}\begin{pmatrix}1&0&2\\0&2&10\\0&0&0\end{pmatrix}\xrightarrow{(1/2)\rho_2}\begin{pmatrix}1&0&2\\0&1&5\\0&0&0\end{pmatrix}$$

These two are row equivalent.

(d) The first,

$$\xrightarrow{\rho_1+\rho_2}\begin{pmatrix}1&1&1\\0&3&3\end{pmatrix}\xrightarrow{(1/3)\rho_2}\begin{pmatrix}1&1&1\\0&1&1\end{pmatrix}\xrightarrow{-\rho_2+\rho_1}\begin{pmatrix}1&0&0\\0&1&1\end{pmatrix}$$

and the second.

$$\xrightarrow{\rho_1+\rho_2}\begin{pmatrix}2&2&5\\0&3&-1\end{pmatrix}\xrightarrow[(1/3)\rho_2]{(1/2)\rho_1}\begin{pmatrix}1&1&5/2\\0&1&-1/3\end{pmatrix}\xrightarrow{-\rho_2+\rho_1}\begin{pmatrix}1&0&17/6\\0&1&-1/3\end{pmatrix}$$

These are not row equivalent.

One.III.2.12 (a) They have the form

$$\begin{pmatrix}a&0\\b&0\end{pmatrix}$$

where $a, b \in \mathbb{R}$.

(b) They have this form (for $a, b \in \mathbb{R}$).

$$\begin{pmatrix}1a&2a\\1b&2b\end{pmatrix}$$

(c) They have the form

$$\begin{pmatrix}a&b\\c&d\end{pmatrix}$$

(for $a, b, c, d \in \mathbb{R}$) where $ad - bc \neq 0$. (This is the formula that determines when a 2×2 matrix is nonsingular.)

Two.I.1.18 (a) $0 + 0x + 0x^2 + 0x^3$

(b) $\begin{pmatrix}0&0&0&0\\0&0&0&0\end{pmatrix}$

(c) The constant function $f(x) = 0$

Two.I.2.20 By Lemma 2.9, to see if each subset of $\mathcal{M}_{2\times2}$ is a subspace, we need only check if it is nonempty and closed.

(a) Yes, it is easily checked to be nonempty and closed. This is a parametrization.

$$\{a\begin{pmatrix}1&0\\0&0\end{pmatrix}+b\begin{pmatrix}0&0\\0&1\end{pmatrix} \mid a, b \in \mathbb{R}\}$$

By the way, the parametrization also shows that it is a subspace, it is given as the span of the two-matrix set, and any span is a subspace.

(b) Yes; it is easily checked to be nonempty and closed. Alternatively, as mentioned in the prior answer, the existence of a parametrization shows that it is a subspace. For the parametrization, the condition $a + b = 0$ can be rewritten as $a = -b$. Then we have this.

$$\{\begin{pmatrix}-b&0\\0&b\end{pmatrix} \mid b \in \mathbb{R}\} = \{b\begin{pmatrix}-1&0\\0&1\end{pmatrix} \mid b \in \mathbb{R}\}$$

(c) No. It is not closed under addition. For instance,

$$\begin{pmatrix}5&0\\0&0\end{pmatrix}+\begin{pmatrix}5&0\\0&0\end{pmatrix}=\begin{pmatrix}10&0\\0&0\end{pmatrix}$$

is not in the set. (This set is also not closed under scalar multiplication, for instance, it does not contain the zero matrix.)

Two.I.2.22 (a) Yes, solving the linear system arising from

$$r_1\begin{pmatrix}1\\0\\0\end{pmatrix}+r_2\begin{pmatrix}0\\0\\1\end{pmatrix}=\begin{pmatrix}2\\0\\1\end{pmatrix}$$

gives $r_1=2$ and $r_2=1$.

(b) Yes; the linear system arising from $r_1(x^2)+r_2(2x+x^2)+r_3(x+x^3)=x-x^3$

$$\begin{aligned} 2r_2+r_3&=1\\ r_1+r_2\quad&=0\\ r_3&=-1\end{aligned}$$

gives that $-1(x^2)+1(2x+x^2)-1(x+x^3)=x-x^3$.

(c) No; any combination of the two given matrices has a zero in the upper right.