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Show all work. Unjustified answer yields no credit.

1. (6 pts) Find two distinct row echelon forms (REF) of the matrix

$$\begin{bmatrix} 2 & 2 & 2 & 4 \\ 4 & 4 & 5 & 6 \end{bmatrix}$$

2. a. (2pts) Define the notion: a matrix A is nonsingular.

- b. (2pts) Suppose that A is 3×3 nonsingular matrix. Write down its reduced row echelon form (RREF).

1

$$\begin{bmatrix} 2 & 2 & 2 & 4 \\ 4 & 4 & 5 & 6 \end{bmatrix} \xrightarrow{\frac{1}{2}R_1 \rightarrow R_1} \begin{bmatrix} 1 & 1 & 1 & 2 \\ 4 & 4 & 5 & 6 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 0 & 2 & -2 \end{bmatrix}$$

First REF (2p)

Now Bring to RREF

$$\begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 0 & 2 & -2 \end{bmatrix} \xrightarrow{R_1 - R_2 \rightarrow R_1} \begin{bmatrix} 1 & 1 & 0 & 4 \\ 0 & 0 & 2 & -2 \end{bmatrix}$$

RREF - Second REF (3p)

2a. A is $n \times n$ matrix (square matrix) such that
 (2p) the system $A\underline{x} = \underline{0}$ has a unique solution
 $\underline{x} = \underline{0}$ (the trivial solution)

b. A is nonsingular \Leftrightarrow its RREF is I_n .
 So for $n=3$ A must have the RREF:

(2p)

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$