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Show all work. Unjustified answer yields no credit.

1. (2 pts) Let  $V$  be a vector space and  $U$  a subset of  $V$ . What are the conditions for  $U$  to be a subspace of  $V$ ?

2. Determine if the following subsets  $U$  of  $\mathbb{R}^3$  are subspaces. Justify!

- a. (4pts)  $U$  consists of all vectors  $\mathbf{x} = (x_1, x_2, x_3)^T$  satisfying  $4x_1 - x_2 + x_3 = 0$ . (1)
- b. (4 pts)  $U$  consists of all vectors  $\mathbf{x} = (x_1, x_2, x_3)^T$  satisfying  $4x_1^2 - x_2^2 + x_3^2 = 0$ .

1. (i) if  $x, y \in U$  then  $x+y \in U$  (1p)  
(ii) if  $x \in U$  then  $ax \in U$  (1p)

2a suppose  $x = (x_1, x_2, x_3)^T$  and  $y = (y_1, y_2, y_3)^T$  satisfy the equation (1)

Then  $4(x_1+y_1) - (x_2+y_2) + (x_3+y_3) = 0 \Rightarrow x+y \in U$  (2p)

clearly  $4(ax_1) - (ax_2) + ax_3 = 0$  hence  $ax \in U$  (2p)

So  $U$  is a subspace

2(b) Take  $x = (1, 2, 0)^T$  and  $y = (0, 1, 1)^T$ .

Both in  $U$ . But  $x+y = (1, 3, 1)^T \notin U$  as

$4 \cdot 1^2 - 3^2 + 1^2 = 4 - 9 + 1 = -4 \neq 0$  (2p)

$U$  not a subspace.