

Name and uic e-mail address:

Shmuel Friedland

Show all work. Unjustified answer yields no credit.

1. (6 pts) Find all roots of $z^3 + 1 = 0$.

2. (4 pts) Let \mathbb{F} be a field. Assume that $A, B \in \mathbb{F}^{n \times n}$ are similar matrices. Show that A and B have the same characteristic polynomial.

$$1 \quad (z^3 + 1)(z + 1)(z^2 - z + 1) = 0$$

First root $z = -1$, other two roots

$$z^2 - z + 1 = 0 \quad z_{2,3} = \frac{1 \pm \sqrt{1^2 - 4}}{2} = \frac{1 \pm \sqrt{-3}}{2} = \frac{1 \pm i\sqrt{3}}{2}$$

$$\frac{1}{2} \pm i \frac{\sqrt{3}}{2}, \quad z_2 = \frac{1}{2} + i \frac{\sqrt{3}}{2}, \quad z_3 = \frac{1}{2} - i \frac{\sqrt{3}}{2}$$

$$2. \quad B = TAT^{-1} \quad \text{Char pol of } B$$

$$\det(\lambda I - B) = \det(\lambda I - TAT^{-1}) =$$

$$\det(T(\lambda I - A)T^{-1}) = \det(T) \det(\lambda I - A) \det T^{-1}$$

$$\det(T) \det(T^{-1}) = \det(TT^{-1}) = \det I_n = 1$$

$$\text{Hence } \det(\lambda I - B) = \det(\lambda I - A)$$