MCS 421 - Spring 2013, Quiz 5, February 22, 2013, Instructor: S. Friedland

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**Show** all work. **Unjustified** answer yields no credit.

1. (10 points). A bakery sells chocolate, cinnamon, poppy seed and plain doughnuts. At a particular time it has 6 chocolate, 5 cinnamon, 3 poppy seed and 30 plain doughnuts. If a box contains 12 doughnuts, how many different options are there for a box of doughnuts?

6 ch, 5 cin, 3 pop, 30 pl $\qquad x_1+x_2+x_3+x_4=12$ (+)

$A_1$ – $x_1$ (number of ch) $\geq 7$

$A_2$ – $x_2$ (number of cin) $\geq 6$

$A_3$ – $x_3$ (number of pop) $\geq 4$

(No need for $A_4$ since 30 pl can)

$|\bar{A}_1\cap\bar{A}_2\cap\bar{A}_3| = |S| - (|A_1|+|A_2|+|A_3|) + (|A_1\cap A_2|+|A_1\cap A_3|+|A_2\cap A_3|) - |A_1\cap A_2\cap A_3|$

$|S| = \binom{12+4-1}{12} = \binom{15}{12} = \binom{15}{3}$

$|A_1|$ – $(x_1-7)+x_2+x_3+x_4 = 12-7=5$ $\qquad |A_1| = \binom{5+3}{5} = \binom{8}{3}$

$|A_2|$ – $x_1+(x_2-6)+x_3+x_4 = 12-6=6$ $\qquad |A_2| = \binom{6+3}{6} = \binom{9}{3}$

$|A_3|$ – $x_1+x_2+(x_3-4)+x_4 = 12-4=8$ $\qquad |A_3| = \binom{8+3}{8} = \binom{11}{3}$

$|A_1\cap A_2| = \phi$ because $7+6=13$

$A_1\cap A_3$ $\quad (x_1-7)+x_2+(x_3-4)+x_4 = 12-11=1$ $\qquad \binom{1+3}{1} = \binom{4}{1} = 4$

$A_2\cap A_3$ $\quad x_1+(x_2-6)+(x_3-4)+x_4 = 12-10=2$ $\qquad \binom{2+3}{2} = \binom{5}{2} = \frac{5\cdot 4}{2} = 10$

$A_1\cap A_2\cap A_3 = \phi$ since $A_1\cap A_2 = \phi$

$\binom{15}{3} - \left(\binom{8}{3}+\binom{9}{3}+\binom{11}{3}\right) + 4 + 10 = \boxed{164}$

$455 - (56+84+165) + 14 = 150+14$

$140+165=305$

$\binom{8}{3} = \frac{8\cdot 7\cdot 6}{6} = 56$

$\binom{9}{3} = \frac{9\cdot 8\cdot 7}{6} = 3\times 4\times 7 = 84$

$\binom{11}{3} = \frac{11\cdot 10\cdot 9}{6} = 165$

$\frac{15\cdot 14\cdot 13}{2\cdot 3}$ $\quad \frac{13}{35}$ $\quad \frac{65}{455}$ $\quad \frac{11}{55}$