MCS 421 - Spring 2013, Quiz 7, March 15, 2013, Instructor: S. Friedland

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Show all work. **Unjustified** answer yields no credit.

Let $h_n = 3h_{n-1} - 2h_{n-2} + 1$.

1. (5 pts) Find the general solution of the above recurrence equation.

2. (5 pts) Let $h_0 = 1, h_1 = 2$. Find the generating function associated with the above recurrence equation.

1. Homogeneous equation $h_n = 3h_{n-1} - 2h_{n-2}$

Char. equation $r^2 = 3r - 2 \Rightarrow r^2 - 3r + 2 = (r-1)(r-2)$

$r_1 = 1 \quad r_2 = 2$ hom. part is $(1)^n = 1$

Hence to guess a particular solution $h_n = An$

$An = 3A(n-1) - 2A(n-2) + 1 \Rightarrow$

$0 = -3A + 4A + 1 = 0 \qquad A = -1 \qquad h_{n,p} = -n$

General solution of homogeneous $a2^n + b$

So $h_n = a2^n + b - n$.

2. First way $h_0 = 1 = a + b \qquad h_1 = 2 = 2a + b - 1$

$2 = (a+b) - 1 + a = 1 - 1 + a \qquad a = 2 \qquad b = -1$

$h_n = 2 \cdot 2^n - 1 - n$

$\frac{1}{1-2x} = \sum_{n=0}^{\infty} 2^n x^n \qquad \frac{1}{1-x} = \sum_{n=0}^{\infty} 1 \cdot x^n$

$\left(\frac{1}{1-x}\right)' = \frac{1}{(1-x)^2} = \sum_{n=0}^{\infty} n x^{n-1} \qquad \frac{x}{(1-x)^2} = \sum_{n=0}^{\infty} n x^n$

$g(x) = \frac{2}{1-2x} - \frac{1}{1-x} - \frac{x}{(1-x)^2}$

Second way $g(x) = \sum_{n=0}^{\infty} h_n x^n \Rightarrow (2x^2 - 3x + 1)g(x) = \frac{x^2}{1-x} + 1 - x$

$g(x) = \frac{1}{(1-2x)(1-x)^2}\left[x^2 + (1-x)^2\right]$