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(4pts) 1.a n linearly independent vectors

(b.) $A = \begin{bmatrix} v_1 & v_2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ $\xrightarrow{R_2 - 2R_1 \to R_2}$ $\begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$ $\xrightarrow{R_2 \to -R_2}$ $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

8pts Two pivots, rank $A = 2$, v_1, v_2 lin. ind.

Solve the system corresponding

$\left[\begin{array}{cc|c} 1 & 2 & 3 \\ 2 & 3 & 4 \end{array}\right]$ ~~$R_2 - 2R_1 \to R_2$~~ $\begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -2 \end{bmatrix}$ $R_1 + 2R_2 \to R_1$

$-R_2 \to R$ $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \end{bmatrix}$ $R_1 - 2R_2$ $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \end{bmatrix}$

(8pts) $x_1 = -1 \quad x_2 = 2 \quad (3,4)^T = -v_1 + 2v_2$

Problem 2 (3p) a. Bring A to a REF

$R_2 + R_1$ $\begin{bmatrix} 1 & 2 & -3 \\ -1 & -1 & 4 \\ 0 & 1 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & -3 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ $R_3 - R_2$ $\to \begin{bmatrix} 1 & 2 & -3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} = U$

a basis in the row space $(1,2,-3), (0,1,1)$ (the two nonzero rows in U. Lead variables x_1, x_2. Hence columns 1 and 2 form a basis in the column space of A

$\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$

(b) rank $A = 2$ null $n = 1$

\# number of lead var.   number of free variables

c. No! rank $I_3 = 3$ while rank $A = 2$

(3pts) The ERO preserves the rank of A.

Problem 3 a $\underline{X} \mapsto A\underline{X}$ where $A \in \mathbb{R}^{n \times n}$ (for some matrix A) $\Leftrightarrow$ $T(\underline{X}) = A\underline{X}$ for some A.

b. $T \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix} \Rightarrow$ $a + b = 2$, $c + d = -1$

$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \Rightarrow$ $b = -1$, $d = 1$

$a = 2 - b = 2 + 1 = 3$ $\quad c = -1 - d = -2$

$$A = \begin{bmatrix} 3 & -1 \\ -2 & 1 \end{bmatrix}$$

Problem 4. a. $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = A^T = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$

$\Rightarrow$ $b = c$ $c = b$. So any symmetric matrix is of the form

$$A = \begin{bmatrix} a & b \\ b & d \end{bmatrix} = a \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}}_{A_1} + b \underbrace{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}}_{A_2} + c \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}}_{A_3}$$

$= \text{span}(A_1, A_2, A_3)$. Hence S_2 is a subspace of $\mathbb{R}^2$.

$aA_1 + bA_2 + dA_3 = 0 \Rightarrow a = b = d = 0$ so A_1, A_2, A_3 lin. ind.

hence $\dim S_2 = 3$

b. $A^T = \begin{bmatrix} a & c \\ b & d \end{bmatrix} = -A = \begin{bmatrix} -a & -b \\ -c & -d \end{bmatrix}$

$\Rightarrow$ $a = -a \Rightarrow a = 0$ $\quad d = -d \Rightarrow d = 0$ $\quad c = -b$ $\quad b = -c$

So $AS_2 = \begin{bmatrix} 0 & -c \\ c & 0 \end{bmatrix} = c\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$= \text{span}\left(\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\right)$ a subspace spanned

by one nonzero vector. So $\dim AS_2 = 1$.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \underbrace{\begin{bmatrix} a & \frac{b+c}{2} \\ \frac{b+c}{2} & d \end{bmatrix}}_{S_2?} + \underbrace{\begin{bmatrix} 0 & \frac{b-c}{2} \\ \frac{c-b}{2} & 0 \end{bmatrix}}_{AS_2}$$

So $\mathbb{R}^{2\times 2} = S_2 + AS_2$

C is symmetric and antisymmetric. So

$a = d = 0$ $\begin{bmatrix} 0 & -c \\ c & 0 \end{bmatrix}$ and $-c = c$

$c = 0$

Hence $S_2 \cap AS_2 = \{0\}$. Hence

$\mathbb{R}^{2\times 2} = S_2 \oplus AS_2$

Problem 5 (a) If V has 5 lin. ind. vectors then $\dim V \geq 5$

(b) $\dim U + \dim V - \dim(U \cap V) = \dim(U+W)$

$3 + 4 - 2 = 5$

(c) A 3×5 AB defined so B $5\times l$

BA defined so B $m\times 3$

Hence B is 5×3

GRADE DISTRIBUTION: $\underbrace{55, 57, 59}_{C-}, \underbrace{63, 66}_{C}, \underbrace{72, 74}_{B-}, \underbrace{91, 96, 100}_{A}$