Linear Algebra II
Summary of Lectures

MATH 425 Linear Algebra II, Spring 2012
LCD-undergrad 24908; LCD-grad 24909,
MWF 10:00-10:50, Addams Hall 303

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1 Week 1: 1/9-1/13, 2012

1.1 January 9

Discussed commutative groups, rings and fields. Started to discuss vector spaces. Pages 1-3 in [2]. For more information on fields see [4, p'138] or
http://en.wikipedia.org/wiki/Field_(mathematics)

1.2 January 11

Discussed notions of vector space, subspace, linear dependence, independence, finitely generated vector spaces, basis. See p'3 in [2], pages 38-51 in [1].

1.3 January 13

The notion of dimension of vector space. See pages 3-5 in January 16, 2012 version of my notes.

2 Week 2: 1/18-1/20, 2012

2.1 January 18

Continued to discussed notions of basis and dimension: pages 3-5 in [2], (version dated January 16, 2012). Started to discuss matrices §1.3 p'6. For more details see my notes [1],pages 7-14, 22-30, on REF and RREF and how to solve the systems of equations.

2.2 January 20

Discussed invertible matrices §1.3.3 in [2]. Discussed row and column spaces of A §1.3.4 [2].

3 Week 3: 1/23-1/27, 2012

3.1 January 23

Covered §1.3.5 and §1.4.1 in [2]. Finished the proof of Theorem 1.10 in §1.4.2 in [2].

3.2 January 25-27

Proved Theorem 1.11. Started the proof of Theorem 1.12. Did parts 1-7.

4 Week 4: 1/30-2/3, 2012

4.1 January 30

Finished section 1.5.

4.2 February 1

Finished §1.6. Started §1.8.

4.3 February 3

Finished §1.8. Started to discuss inner product spaces over $\mathbb{R}$. Used my notes [1], pages 113-121.

5 Week 5: 2/6-2/10, 2012

5.1 February 6

Did pages 120–130 in [1].

5.2 February 8

Orthonormal sets, Bessel inequality, Parseval identity, Gram-Schmidt process, QR algorithm. Pages 131–135, 138–141, 143 in [1].

5.3 February 10

Inner product space over reals. Pages 145–147 in [1]. Inner product space over complex numbers. Example $\mathbb{C}^n$, $< \mathbf{u}, \mathbf{v} >= \mathbf{v}^*\mathbf{u}$. General properties of inner product space over complex numbers. Proves Cauchy-Schwarz inequality. Pages 18 and Problem (2.3) parts (a), (b) on page 20 in [2].

6 Week 6: 2/13-2/17, 2012

6.1 February 13

Solved quiz 3. Did pages 18–19 in [2]. Explained and proved the geometric interpretation of the determinant. Proved the Hadamard determinantal inequality. Pages 22–23 in [2].

6.2 February 15

Discussed special transformation in inner product spaces as self-adjoint, symmetric, antiself-adjoint, skew-symmetric, unitary, orthogonal. Proved Theorem 2.17. Pages 23–25 in [2]. Discussed the notion of complexification of real inner product space, page 18.

6.3 February 17

Discussed real symmetric matrices as a special case of hermitian matrices, real orthogonal matrices as a special case of unitary matrices. Read pages 218–229 in [1]. Discussed Proposition 2.22 in [2, page 26]. (Will discuss the proof in the next class.) Discussed Euler's theorem for orthogonal rotations in $\mathbb{R}^3$.

http://en.wikipedia.org/wiki/Euler%27s_rotation_theorem

http://mathworld.wolfram.com/RotationMatrix.html

Proved Schur's theorem: Corollary 2.24 [2, page 27].

Gave quiz 4.

7 Week 7: 2/20-2/24, 2012

7.1 February 20

Solved quiz 4. Restated Proposition 2.21 as follows: Let $Q \in \mathbb{R}^{n\times n}$ be an orthogonal matrix. Then there exists an orthogonal matrix $T \in \mathbb{R}^{n\times n}$ such that the following holds. $T^\top QT$ is a block diagonal orthogonal matrix of the following form. The 1×1 diagonal blocks are either 1 or -1. The 2×2 diagonal blocks are plane rotation by the angle $\theta \in (-\pi, \pi)$. That is, it is of the form $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$.

7.2 February 22

Solved two problems from HW 5. Started to discuss §2.5. Pages 31–32 in [2], upto Corollary 2.37.

7.3 February 24

Did Problems 1 and 5 on page 23 in [2]. Explained and proved Corollary 2.37 and Theorem 2.42 on pages 32 and 33 in [2]. Gave quiz 5.

8 Week 8: 2/27-3/02, 2012

8.1 February 27

Solved quiz 5. Proved Courant–Fischer characterization Theorem 2.38. Stated and proved Theorem 2.40.

8.2 February 29

Pages 36–37 in [2].

8.3 March 2

Finish the proof of the formula (2.2) for the determinant of Gramian in [2]. proved Corollary 2.49. Started to prove Corollary 2.51. Discussed the LU decomposition, see pages 235–241 in [1].

9 Week 9: 3/05-3/09, 2012

9.1 March 5

Solved quiz 6. Discussed LU decomposition, $LDL^\top$ decomposition and Cholesky decompositiiton. See pages 235–250 in [1]. proved Corollary on page 252 in [1]. Discussed quadratic forms pages 230–232 in [1]. Stated Sylvester's law of inertia. If $B = SAS^\top$ where A, B symmetric and S invertible, then A and B have the same number of positive, negative and zero eigenvalues.

9.2 March 7

Gave Midterm.

9.3 March 9

Solved the midterm and gave the grade distribution.

http://homepages.math.uic.edu/∼friedlan/solmidM425S12.pdf

Started to discuss the singular value decomposition - SVD. See page 263 in [1].

10 Week 10: 3/12-3/16, 2012

10.1 March 12

Continued to discuss SVD and its approximation properties. Pages 263–277 in [1]. See also pages 42–45 in [2]. Proved the approximation properties of A_p to A given in Theorem 2.63 [2].

Stated the right-hand side of the inequality (2.1) on page 40 in [2].

10.2 March 14

Proved Theorem 2.55, Theorem 2.66, Theorem 2.73, Theorem 2.74, Corollary 2.75.

10.3 March 16

Discussed Moore-Penrose inverse §2.10 in [2], pages 52–54. Discussed the representation problem of a similarity class to given matrix $A \in \mathbb{C}^{n \times n}$. Pages 54–57 in [2], not including Corollary 3.11. See pages 194–204 in [1].

Gave quiz 7.

11 Week 11: 3/26-3/30, 2012

Pages 59–62 up to Proposition 3.20.

12 Week 12: 4/2-4/6, 2012

12.1 April 2

Solved quiz Quiz 8. Pages 62–63 in [2].

12.2 April 4

Finished the proof of Theorem 3.23 in [2], i.e. pages 63–64. Use Theorem 3.23 over $\mathbb{C}$ to show that we can choose $\psi_j = (z - \lambda_j)$ for $j = 1, \ldots, k$, where $\lambda_1, \ldots, \lambda_k$ are all pairwise distinct eigenvalues of T. Hence, to find a nice representative of the similarity class of $A\mathbb{C}^{n\times n}$, it is enough to consider the case of a nilpotent matrix A, i.e. $A^n = 0$.

Continued from Herstein's lectures [5] pages 293–294. Formulated Lemma 6.5.3, but did not prove it.

12.3 April 6

Proved Lemmas 6.5.3, 6.5.4 and Theorem 6.5.1 in [5], pages 294–296.

13 Week 13: 4/09-4/13, 2012

13.1 April 9

Finished the proof Lemmas 6.53, proved Lemma 6.54 and the decomposition on top of the page 296. Proved Theorem 6.6.2 on page 301. Gave quiz 9.

13.2 April 11

Introduced $s_j(A)$ (3.7) on page 66 in [2]. Proved Theorem 3.26, Corollary 3.27, Theorem 3.13 and Corollary 3.29 pages 66–69 in [2].

13.3 April 13

Did pages 75–78 in [2].

14 Week 14: 4/16-4/20, 2012

14.1 April 16

Showed that the components of A are linearly independent matrices. Did pages 79–80 in [2] up to Corollary 4.7. Gave quiz 10

14.2 April 18

Solved quiz 10. Discussed Corollary 4.7. Did page 82 and first part of Proposition 4.10. Proved formula (4.14). Discussed the notions of stability given in Definition 4.11 on page 83.

14.3 April 20

Stated and proved Theorem 4.12 and Corollary 4.13 on pages 83–84. Started to discuss the notion of cyclic subspaces and the minimal polynomial of A with respect to a nonzero vector $\mathbf{u}$. Page 70 of [2].

15 Week 15: 4/23-4/27, 2012

15.1 April 23

Page 71 of [2].

References

[1] S. Friedland, Outline of Lectures in Linear Algebra Math 320, http://homepages.math.uic.edu/∼friedlan/math320lecS12.pdf

[2] S. Friedland, Linear Algebra II, Lectures Notes, http://homepages.math.uic.edu/∼friedlan/lectnotesM425S12.pdf

[3] G.H. Golub and C.F. Van Loan. Matrix Computation, *John Hopkins Univ. Press, 3rd Ed.*, Baltimore, 1996.

[4] J. Hefferon, *Linear Algebra*, http://joshua.smcvt.edu/linearalgebra/Linear Algebra

[5] I.N. Herstein, *Topics in Algebra*, John Wiley & Sons, 1975.

[6] S.J. Leon, *Linear Algebra with Applications*, Prentice Hall, 6th Edition, 2002.

[7] S. Lipschutz and M. Lipson, *Linear Algebra*, Fourth Edition, Schaum's Outlines, McGraw-Hill, 2009.

[8] P. Petersen, *Linear Algebra*, http://www.math.ucla.edu/∼petersen/linalg3.pdf