Linear Algebra II
Summary of Lectures

MATH 425 Linear Algebra II, Spring 2015
LCD-undergrad 24908; LCD-grad 24909,
MWF 10:00-10:50, Thaft Hall 313

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Last update April 29, 2015

1 Week 1: 1/12-1/16, 2015

1.1 January 12

Discussed commutative groups, rings and fields. Started to discuss vector spaces. Pages 1-5 in [2]. (Started §1.2.1 on page 5.) For more information on fields see [4, p'138] or

http://en.wikipedia.org/wiki/Field_(mathematics)

1.2 January, 14

Finished 1.2: pages 5-7.

1.3 January 16

Finished §1.2.1, §1.2.2,§1.3.1-1.3.3.

2 Week 2: 1/21-1/23, 2015

2.1 January 21

Did §1.3.4. Gave quiz 1.

2.2 January 23

Did §1.3.5, §1.4.1, §1.4.2. Started §1.5. Stopped after the Example 1.

3 Week 3: 1/26-1/30, 2015

3.1 January 26

Finished §1.5. Started §1.6. Proved *1-5* of Theorem 1.12 in §1.6. Started to prove *6*. Showed that $\det E \neq 0$ and $\det EA = (\det E)(\det A)$ for an elementary matrix A.

3.2 January 28

Finished the proof of Theorem 1.12. Started to prove Proposition 1.13. Gave quiz 2.

3.3 January 30

Solved quiz 2. Finished the proof of Proposition 1.13. (It is now Proposition 1.14 in the new version.) Proved Poposition 1.15 (in the new version.) Covered §1.7. Skipped §1.7.1. Skipped §1.8. (It is assumed that students know it. If not please refresh your knowledge.) Started §1.9. Discussed the formulas of change of bases, in particular proved (1.30). Mention that if $\mathbf{U} = \mathbf{V}$ and the bases in $\mathbf{V}$ are the same: $[\mathbf{u}_1, \dots, \mathbf{u}_m] = [\mathbf{v}_1, \dots, \mathbf{v}_n]$, $(m = n)$, then th representation matrix A changes to $X^{-1}AX$.

4 Week 4: 2/2-2/6, 2015

4.1 February 2, 2015

Finished §1.9. Started §2.1 Did Lemma 2.1 and Corollary 2.2. (See the new version of notes uploaded today).

4.2 February 4, 2015

Covered from Corollary 2.2 to Corollary 2.7, pages 24-25 in my notes. Gave quiz 3.

4.3 February 6

Finished §2.1. Started §2.2. Finished at formula (2.1).

5 Week 5: 2/9-2/13, 2015

5.1 February 9

Finished §2.2. Started §2.3. Stated Theorem 2.19.

5.2 February 11

Proved Theorem 2.19, did Prop. 2.20 and Def. 2.21. Gave quiz 4.

5.3 February 13

Discussed §2.4 Started to prove Theorem 2.19.

6 Week 6: 2/16-2/20, 2015

6.1 February 16

Finished the proof of the existence of JCF. Discussed Definition 2.26. Proved the uniqueness of the JCF. Did Corollary 2.29 and Definition 2.30.

6.2 February 18

Finished §2.5. Started §2.6. Discussed the notion of a cyclic subspace, page 39 of [2]. Gave quiz 5.

6.3 February 20

Did Lemma 2.32, Theorem 2.33. Stated Theorem 2.34. Started to prove it.

7 Week 7: 2/23-2/27, 2015

7.1 February 23

Finished the proof of Theorem 2.34. Proved Theorem 2.35. Stated Lemma 2.36. Will prove later. Proved Propositions 2.37 and 2.38.

7.2 February 25

Proved Lemma 2.41. Completed the proof of Theorem 2.40. Gave quiz 6.

7.3 February 27

Proved Cauchy-Binet fromula: Lemma 2.36. Started §3.1. Stated and proved Theorem 3.1.

8 Week 8: 3/2-3/6, 2015

8.1 March 2

Stated and proved Theorem 3.2 and Proposition 3.3. Gave Definition 3.5. Stated Theorem 3.6.

8.2 March 4

Proved Theorem 3.6. Defined e^A, e^{At}. Showed (3.13) for $\mathbf{b} = \mathbf{0}$. Gave Definition 3.11. Stated Theorem 3.12.

8.3 March 6

Gave an algorithm to find the components of A, pages 47 - 48 in the current version. Discussed (3.14). Proved Theorem 3.12 and Corollary 3.13.

9 Week 9: 3/9-3/13, 2015

9.1 March 9

Started §4.1. Finished Definition 4.4. Discussed the geometrical meaning of Gram-Schmidt process.

9.2 March 11

Did an example for Gram-Schmidt process (page 58 of my notes). Explained the QR decomposition. Did an example (page 59.) Explained QR algorithm. (Not in the notes.)

9.3 March 13

Finished §4.5 and §4.6.

10 Week 10: 3/16-3/20, 2015

10.1 March 16

Started §4.7. Proved Proposition 4.16. Did pages 64 and 65.

10.2 March 18

Solved the midterm.

10.3 March 20

Stated and proved Theorem 4.21. Stated and proved Corollary 4.22 (in the last version of lecture notes of March 21, 2015.) Stated and proved Proposion 4.24.

11 Week 11: 3/30-4/3, 2015

11.1 March 30

Did Proposition 4.25, Theorem 4.27 and Corollary 4.28. Started to discuss Theorem 4.30.

11.2 April 1

Finished the proof of Theorem 4.30. FInished §4.8. Started §4.9. Proved the Rayleigh maximum characterization of $\lambda_1(T)$ and the minimum characterization of $\lambda_n(T)$ - formula (4.2). Stated the formula (4.1).

11.3 April 3

Proved Theorem 4.40, Corollary 4.41. Stated Theorem 4.42. Gave quiz 8.

12 Week 12: 4/6-4/10, 2015

12.1 April 6

Shiwed that equlaity case in Problem 2 on page 64 is achieved for the matrix $A = [a_{pq}] \in \mathbb{C}^{n\times n}$ if $a_{pq} = \zeta^{(p-1)(q-1)}$, where ζ is a primitive root of unity, e.g. $\zeta = e^{\frac{2\pi i}{n}}$. Proved Theorem 4.42, Corollary 4.43, Theorem 4.44. Stated Theorem 4.46.

12.2 April 8

Proved Theorem 4.46. Stated Theorem 4.47. Started §4.10. Did Definition 4.48, Corollary 4.49, Proposition 4.50.

12.3 April 10

Defined Grammian. Stated Tehorem 4.52. Covered the Proof of Theorem 4.52 upto and including (4.2)

13 Week 13: 4/13-4/17, 2015

13.1 April 13

Finished the proof Theorem 4.52, Corollary 4.53 and Theorem 4.54. Solved Quiz 9.

13.2 April 15

Did Proposition 4.56 for hermitian matrices. Stated and proved Theorem 4.59.

13.3 April 17

Did Corollary 4.60. Did pages 83-84. Stated the best approximation property related to SVD given by Corollary 4.79.

14 Week 14: 4/20-4/24, 2015

14.1 April 20

Stated and proved Theorem 4.67. Stated and proved Theorem 4.70. Stated Theorem 4.77 for matrices.

14.2 April 22

Proved Theorem 4.77 for matrices. Proved Theorem 4.78 for matrices. Gave quiz 10.

14.3 April 24

Solved quiz 10. Proved Theorem 4.82. Finished §4.14.

15 Week 15: 4/27-5/1, 2015

15.1 April 27

Started Chapter 5. Did pages 95-97 until Lemma 5.5

15.2 April 29

Did Lemmas 5.5, 5.6, Theorem 5.7 and their proof. Proved parts 1 and 2 of Theorem 5.1.

References

[1] S. Friedland, Outline of Lectures in Linear Algebra Math 320, http://homepages.math.uic.edu/∼friedlan/math320lecS12.pdf, http://homepages.math.uic.edu/∼friedlan/math320lecS13t.pdf

[2] S. Friedland, Linear Algebra II, Lectures Notes, http://homepages.math.uic.edu/∼friedlan/lectnotesM425S15.pdf

[3] G.H. Golub and C.F. Van Loan. Matrix Computation, *John Hopkins Univ. Press, 3rd Ed.*, Baltimore, 1996.

[4] J. Hefferon, *Linear Algebra*, http://joshua.smcvt.edu/linearalgebra/

[5] I.N. Herstein, *Topics in Algebra*, John Wiley & Sons, 1975.

[6] R.J. Horn and C.R. Johnson, *Matrix Analysis*, Cambridge University Press, 2ed, 2013.

[7] S.J. Leon, *Linear Algebra with Applications*, Prentice Hall, 6th Edition, 2002.

[8] S. Lipschutz and M. Lipson, *Linear Algebra*, Fourth Edition, Schaum's Outlines, McGraw-Hill, 2009.