MCS 421 -Combinatorics, **Midterm**,
Spring 2013, Instructor: S. Friedland
Name and e-mail address:

No books or notes. Show all your work. Write solutions in the exam booklet without copying the problems. You can use a result (x) of any part of the problem, to show other part of any problem. **Unjustified** answer yields no credit.

Problem 1. I have 3 red balls, 3 blue balls and 4 black balls. (The balls of the same color are indistinguishable.)
a. (10pts) I have 9 children at my home. I give each child 1 ball. What is the number of all possible choices?
b. (10pts) I pack a number of balls into a suitcase. I take at least one ball of each color. What is the number of choices?
Problem 2. In my house I have 5 weights, whose weights are i_1, i_2, i_3, i_4, i_5 pounds, where i_1, i_2, i_3, i_4, i_5 are positive integers.
a. (15pts) Assume that the total weight of these 5 weights is 30. Show that I could choose a number of weights to hold in my left and right arm simultaneously, so that each of my arms lifts the same total weight.
b. (10pts) Assume that the total weight of these weights is 31. Give an example of weights for which part a of the problem does hold.
Problem 3.
a. (15p.) Let n, k be nonnegative integers and $n \geq k$. Prove the identity

$$\binom{n+1}{k+1} = \binom{0}{k} + \binom{1}{k} + \cdots + \binom{n-1}{k} + \binom{n}{k}.$$

b. (10 p.) In a fitness test on needs to perform the following exercises: bike 10 minutes, speed run 5 minutes, stair master 8 minutes, rowing 7 minutes. One can alternate the exercises in quantum of 1 minute. For example run 2 minutes, row 1 minute, run 3 minutes, row 5 minutes and cetera. How may choices are there?
Problem 4.
a. (15pts) We place 5 rooks on 5×5 board. Let $X_i \subset \{1, 2, 3, 4, 5\}$ be the subset of forbidden position on row i. Assume $X_1 = \{1\}, X_2 = \{1, 2\}, X_3 = X_4 = \{3, 4\}, X_5 = \emptyset$. Draw the 5×5 board with the forbidden positions and find the number allowable placements.
b. (15pts) Determine the probability of having a permutation of $\{1, 2, \ldots, n\}$, for $n \geq 4$, such that exactly 2 integers are in their place.