Try to solve the following challenging problems.
Time has no importance.

Problem 1. Consider a configuration of n non-intersecting balls of unit radius in $\mathbb{R}^3$, representing planets in the space. A point on a surface of a ball is called "isolated" if an observer located at this point will not be able to see any other ball or part of a ball in the sky. Compute the total area of the set of all isolated points in the system.

Problem 2. Prove that for any configuration of $2n$ points $A_1, \ldots, A_n, B_1, \ldots, B_n$ in the plane, no three of which are on the same line, there exists a permutation π of $\{1, \ldots, n\}$ such that the n intervals $[A_i, B_{\pi(i)}]$ are disjoint.

Problem 3. For each letter of the alphabet A, B, ... find the maximal cardinality of a set of disjoint homeomorphic copies of the letter that can be imbedded in the plane. Hint: one can imbed continuum many ($2^{\aleph_0}$) disjoint copies of C or O, but no more than countably many ($\aleph_0$) copies of A or Y.

Problem 4. Let $G = \mathrm{Homeo}_+(\mathbb{R})$ denote the group of all orientation preserving homeomorphisms of the real line. Prove

(1) If $f, g \in G$ have no fixed points, then f is conjugate to either g or g^{-1}.
(2) The subgroup $G_0 < G$, consisting of $g \in \mathrm{Homeo}_+(\mathbb{R})$ satisfying $g(x) = x$ on $(\infty, -c] \cup [c, +\infty)$ for some $c = c(g)$, is simple.

Problem 5. Prove that any function $f : \mathbb{Q} \to \mathbb{Q}$ can be written as $f = f_1 + f_2 + f_3$ where $f_i : \mathbb{Q} \to \mathbb{Q}$ are bijections.

Problem 6. A one meter long stick is lifted from the ground, together with $n = 20$ ants. Ants are in motion, their speed is constant (one meter per minute), initial position on the stick and direction of movement are random. When two ants meet they reverse directions and walk away from each other, reaching an end of the stick ants jump to the ground. Compute the probability $P(t)$ of all ants leaving the stick by time t.

Problem 7. Let X be an undirected graph with n vertices, without double edges or self edges. Suppose that for any two vertices $i \neq j$ there is precisely one vertex $k \neq i, j$ connected to both i and to j. Prove that

(1) either the graph is star-shaped, i.e., one vertex connected to all other vertices, and no other edges are present;
(2) or $n = m^2 + m + 1$ and every vertex is connected to $(m + 1)$ other vertices.

Give an example for (2) when m is a prime or a power of a prime.

Problem 8. Show that a polynomial $f(x) \in \mathbb{R}[x]$ is positive (has $f(x) \geq 0$ or all $x \in \mathbb{R}$) if and *only if* $f(x) = p(x)^2 + q(x)^2$ for some $p, q \in \mathbb{R}[x]$.

Problem 9. Prove that it is impossible to cover the plane by finitely many isometric copies of the interior of a parabola $\{(x, y) \mid y \geq x^2\}$ (shifted and rotated).

Problem 10. Prove that it is impossible to find four points in the plane, so that all six distances between pairs are odd integers.