

**MATH 414. Hour Exam One. Part II (take home). 2010-02-26**

**Problem 1.**

- (a) Give a definition of a **closed** set.
- (b) Give a definition of a **compact** set.
- (c) Prove that any compact set is closed.

**Problem 2.**

Let  $K \subset \mathbf{R}$  be compact. Prove that the set

$$K - K = \{x - y \mid x \in A, y \in K\}$$

is also compact.

**Problem 3.**

Prove that every non-empty open set  $U$  admits a bijection with  $\mathbf{R}$ .

**Problem 4.**

Let  $f : \mathbf{R} \rightarrow \mathbf{R}$  be a non-decreasing function. Suppose that  $f$  maps  $\mathbf{Q}$  to  $\mathbf{Q}$  and  $f : \mathbf{Q} \rightarrow \mathbf{Q}$  bijection (such a map can be complicated). Prove that  $f : \mathbf{R} \rightarrow \mathbf{R}$  is continuous, one-to-one and onto.