Math 180 Hour Exam One Sep. 2007

Solutions

1. $\lim\limits_{x\to 1} \dfrac{x^2+2x-3}{x^2-x} = \lim\limits_{x\to 1} \dfrac{(x+3)(x-1)}{x(x-1)} = \lim\limits_{x\to 1} \dfrac{x+3}{x} = \dfrac{1+3}{1} = 4$

2. (a) $\left(4x^3-5x^{\frac{1}{3}}+3x^{-2}\right)' = (4x^3)' - (5x^{\frac{1}{3}})' + (3x^{-2})'$

$= 4\cdot 3x^2 - 5\cdot\frac{1}{3}x^{-\frac{2}{3}} + 3\cdot(-2)x^{-3}$

$= 12x^2 - \frac{5}{3}x^{-\frac{2}{3}} - 6x^{-3}$

(b) $\left((x^2-3x)e^x\right)' = (x^2-3x)'\cdot e^x + (e^x)'\cdot(x^2-3x)$

$= (2x-3)\cdot e^x + e^x\cdot(x^2-3x)$

(c) $\left(\dfrac{x-3}{x^2+x+1}\right)' = \dfrac{(x-3)'(x^2+x+1)-(x-3)(x^2+x+1)'}{(x^2+x+1)^2}$

$= \dfrac{x^2+x+1-(x-3)(2x+1)}{(x^2+x+1)^2}$

3. $y'|_{x=2} = (x^3-3x)'|_{x=2} = 3x^2-3|_{x=2} = 3\cdot 2^2-3 = 9$

The tangent line is

$$y = 9(x-2)+(2^3-3\cdot 2) = 9(x-2)+2$$

4. $\text{AROC} = \dfrac{f(9)-f(4)}{9-4} = \dfrac{\sqrt{9}-\sqrt{4}}{5} = \dfrac{3-2}{5} = \dfrac{1}{5}$

$\text{IROC} = \lim\limits_{x\to 4}\dfrac{f(x)-f(4)}{x-4} = \lim\limits_{x\to 4}\dfrac{\sqrt{x}-2}{x-4} = \lim\limits_{x\to 4}\dfrac{(\sqrt{x}-2)(\sqrt{x}+2)}{(x-4)(\sqrt{x}+2)}$

$= \lim\limits_{x\to 4}\dfrac{x-4}{(x-4)(\sqrt{x}+2)} = \lim\limits_{x\to 4}\dfrac{1}{\sqrt{x}+2} = \dfrac{1}{\sqrt{4}+2} = \dfrac{1}{4}$.

5. (a) $f'(5) = \lim_{h\to 0} \frac{f(5+h)-f(5)}{h} = \lim_{h\to 0} \frac{\frac{1}{5+h} - \frac{1}{5}}{h}$

(b) $f'(5) = \lim_{h\to 0} \frac{\frac{1}{5+h} - \frac{1}{5}}{h} = \lim_{h\to 0} \frac{\frac{5-(5+h)}{5(5+h)}}{h}$

$= \lim_{h\to 0} \frac{-h}{5(5+h)} \cdot \frac{1}{h}$

$= \lim_{h\to 0} -\frac{1}{5(5+h)}$

$= -\frac{1}{25}$

6. $\frac{f(2.3)-f(2.5)}{2.3-2.5} = \frac{1.41-1.38}{-0.2} = \frac{0.03}{-0.2} = -0.15$

$\frac{f(2.4)-f(2.5)}{2.4-2.5} = \frac{1.40-1.38}{-0.1} = \frac{0.02}{-0.1} = -0.2$

$\frac{f(2.6)-f(2.5)}{2.6-2.5} = \frac{1.35-1.38}{0.1} = \frac{-0.03}{0.1} = -0.3$

$\frac{f(2.7)-f(2.5)}{2.7-2.5} = \frac{1.31-1.38}{0.2} = \frac{-0.07}{0.2} = -0.35$

$f'(2.5) \approx \frac{(-0.2)+(-0.3)}{2} = -0.25$ (It is the average of -0.2 and -0.3)

The reason to choose the secant line passing through $(2.5, f(2.5))$ is $f'(2.5) = \lim_{x\to 2.5} \frac{f(x)-f(2.5)}{x-2.5}$.