Math 180 Sample problems for Hour Exam One

Solutions

1. $f(x)$ is linear. The slope of a linear function is given by $m = \frac{f(x_1) - f(x_2)}{x_1 - x_2}$, in other words $m = \frac{y_1 - y_2}{x_1 - x_2}$

Then the function $f(x) = \frac{f(x_1) - f(x_2)}{x_1 - x_2}(x - x_1) + f(x_1)$

Indeed, this is just the slope-point form of a linear function. In this particular case

$$m = \frac{4 - 2}{4 - 1} = \frac{2}{3}$$

$$f(x) = \frac{2}{3}(x - 1) + 2$$

So $f(7) = \frac{2}{3}(7-1) + 2 = \frac{2}{3} \cdot 6 + 2 = 4 + 2 = 6$

2. (When you see quadric expressions in a fraction, if possible, try to simplify the fraction by factoring the quadrics and cancelling common factors.)

$$\lim_{x \to 2} \frac{x^2 - 3x + 2}{x^2 - x - 2} = \lim_{x \to 2} \frac{(x-1)(x-2)}{(x+1)(x-2)} = \lim_{x \to 2} \frac{x-1}{x+1} = \frac{2-1}{2+1} = \frac{1}{3}$$

3. (A function $f(x)$ is continuous at a point $x = a$ if and only if $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a)$.)

The answer for this problem is $x = 4$, since $\lim_{x \to 4^-} f(x) = 1$ $\lim_{x \to 4^+} f(x) = 2$.

4. (a) $(x^3+x^{\frac{1}{3}})' = (x^3)' + (x^{\frac{1}{3}})' = 3x^{3-1} + \frac{1}{3}x^{\frac{1}{3}-1}$

$= 3x^2 + \frac{1}{3}x^{-\frac{2}{3}}$

(b) $(x^2 e^x)' = (x^2)' \cdot e^x + (e^x)' \cdot x^2$

$= 2x \cdot e^x + e^x \cdot x^2$

(c) $\left(\frac{2+x}{3+x^2}\right)' = \frac{(2+x)'(3+x^2) - (3+x^2)'(2+x)}{(3+x^2)^2}$

$= \frac{(x)'(3+x^2) - (x^2)'(2+x)}{(3+x^2)^2}$

$= \frac{(3+x^2) - 2x(2+x)}{(3+x^2)^2}$

5. (a) $(1+x+\frac{1}{2}x^2+\frac{1}{6}x^3)' = (x)' + (\frac{1}{2}x^2)' + (\frac{1}{6}x^3)'$

$= 1 + x + \frac{1}{2}x^2$

(b) $((x-1)e^x)' = (x-1)' \cdot e^x + (e^x)' \cdot (x-1)$

$= 1 \cdot e^x + e^x \cdot (x-1) = e^x + e^x \cdot (x-1)$

(c) $\left(\frac{1}{\sqrt{x}-1}\right)' = \frac{(1)' \cdot (\sqrt{x}-1) - 1 \cdot (\sqrt{x}-1)'}{(\sqrt{x}-1)^2} = \frac{0 - (\sqrt{x}-1)'}{(\sqrt{x}-1)^2}$

$= -\frac{(\sqrt{x})'}{(\sqrt{x}-1)^2} = -\frac{\frac{1}{2}x^{-\frac{1}{2}}}{(\sqrt{x}-1)^2} = -\frac{1}{2\sqrt{x}(\sqrt{x}-1)^2}$

6. (Given a differentiable function $f(x)$, the tangent line of $y=f(x)$ at $x=a$ is given by

$y = f'(a)(x-a) + f(a)$.

)

In this problem, the tangent line is

$$y = f'(2)(x-2) + f(2)$$

Need to compute $f'(x)$ at first.

$$f'(x) = (-x^2+7x)' = -2x+7$$

So $f'(2) = -2\cdot 2+7 = 3$. $f(2) = -2^2+7\cdot 2 = 10$

Hence $y = f'(2)(x-2) + f(2) = 3(x-2)+10$

7. (Given a function $f(x)$. the AROC ~~over~~ on an interval $[a, b]$ is defined by $\frac{f(b)-f(a)}{b-a}$. the IROC at $x=a$ is defined by $\lim_{x\to a} \frac{f(x)-f(a)}{x-a}$. This is just the definition of the derivative of $f(x)$ at $x=a$.)

(a) $$\frac{f(3)-f(1)}{3-1} = \frac{(3^2+3)-(1^2+3)}{2} = \frac{12-4}{2} = \frac{8}{2} = 4$$

(b) $$\lim_{x\to 2}\frac{f(x)-f(2)}{x-2} = \lim_{x\to 2}\frac{(x^2+3)-(2^2+3)}{x-2} = \lim_{x\to 2}\frac{x^2-2^2}{x-2}$$

$$= \lim_{x\to 2}\frac{(x-2)(x+2)}{x-2} = \lim_{x\to 2}(x+2) = 4$$

8. $$f'(3) = \lim_{x\to 3}\frac{f(x)-f(3)}{x-3} = \lim_{x\to 3}\frac{x^2-3^2}{x-3} = \lim_{x\to 3}(x+3) = 6$$

9. (Slope of a secant line passing through $(a, f(a))$ and $(b, f(b))$ is defined by

$$\frac{f(b)-f(a)}{b-a}$$)

$$\frac{f(1.8)-f(2)}{1.8-2} = \frac{2.24-2.30}{1.8-2} = \frac{-0.06}{-0.2} = 0.3$$

$$\frac{f(1.9)-f(2)}{1.9-2} = \frac{2.27-2.30}{1.9-2} = \frac{-0.03}{-0.1} = 0.3$$

$$\frac{f(2.1)-f(2)}{2.1-2} = \frac{2.33-2.30}{2.1-2} = \frac{0.03}{0.1} = 0.3$$

$$\frac{f(2.2)-f(2)}{2.2-2} = \frac{2.37-2.30}{2.2-2} = \frac{0.07}{0.2} = 0.35$$

So $f'(2) \approx 0.3$

10. (For $\frac{0}{0}$ type limit, try to multiply the conjugate to both top and bottom of the fraction.)

$$\lim_{x\to 1} \frac{\sqrt{3x+1}-\sqrt{2x+2}}{x-1} = \lim_{x\to 1} \frac{(\sqrt{3x+1}-\sqrt{2x+2})(\sqrt{3x+1}+\sqrt{2x+2})}{(x-1)(\sqrt{3x+1}+\sqrt{2x+2})}$$

$$= \lim_{x\to 1} \frac{3x+1-(2x+2)}{(x-1)(\sqrt{3x+1}+\sqrt{2x+2})} = \lim_{x\to 1} \frac{x-1}{(x-1)(\sqrt{3x+1}+\sqrt{2x+2})}$$

$$= \lim_{x\to 1} \frac{1}{\sqrt{3x+1}+\sqrt{2x+2}} = \frac{1}{\sqrt{3+1}+\sqrt{2+2}} = \frac{1}{2+2} = \frac{1}{4}$$