Math 180 **Hour Exam Two** **November 2007**

For every question, write out your computations in the exam booklet.

1. Find the derivative of the following functions, do simplify.

(a) $\ln(x^2 + x + 1)$, (b) $\cos(\sqrt{x})$, (c) $\arctan(x)$.

2. Find the derivatives $f'(x)$ and $f''(x)$ for the function $f(x) = e^{-x}\sin(x)$.

3. Use implicit differentiation to find the slope of the line tangent to the curve

$$xy^2 + 2x^2 - y = 0$$

at the point $(-1, 1)$.

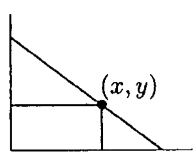
4. Let $f(x) = x^4 - 6x^2 + 4$.
   (a) Find the critical points and the inflections points of f.
   (b) On what interval is f concave down?
   (c) Find the minimum value of f?

5. Find the limit: $\lim\limits_{x\to 1} \dfrac{\ln(x)}{x^3 - 1}$.

6. A family of rectangles in the xy-plane has one side on the x-axis, the lower left corner at the origin $(0, 0)$, and the upper right corner at a point (x, y) on the straight line

$$3x + 4y = 5.$$

   (a) Find the area of such a rectangle as a function of x alone.
   (b) Find the dimensions, x and y, of the particular rectangle with the largest area.



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1 a) $\left(\ln(x^2+x+1)\right)' = \frac{1}{x^2+x+1}\cdot(x^2+x+1)' = \frac{2x+1}{x^2+x+1}$

b) $\left(\cos(\sqrt{x})\right)' = -\sin(\sqrt{x})\cdot(\sqrt{x})' = -\frac{\sin(\sqrt{x})}{2\sqrt{x}}$

c) $(\arctan x)' = \frac{1}{1+x^2}$

2. $f'(x) = (e^{-x}\sin x)' = -e^{-x}\sin x + e^{-x}\cos x = e^{-x}(\cos x - \sin x)$

$f''(x) = -e^{-x}(\cos x - \sin x) + e^{-x}(-\sin x - \cos x) = -2e^{-x}\cos x$

3. $(xy^2+2x^2-y)' = 0 \Rightarrow (xy^2)' + (2x^2)' - y' = 0$

$\Rightarrow y^2 + x\cdot 2yy' + 4x - y' = 0$

$\Rightarrow y^2+4x + (2xy-1)y' = 0$

$\Rightarrow y' = -\frac{y^2+4x}{2xy-1}$

$y'\Big|_{(-1,1)} = -\frac{1^2+4(-1)}{2(-1)\cdot 1 - 1} = -1$

$\Rightarrow$ tangent line

$y-1 = -1\cdot(x-(-1)) \Leftrightarrow y = -x$

4. $f'(x) = 4x^3-12x = 0 \Rightarrow 4x(x^2-3)=0 \Rightarrow x=0$ or $x=\pm\sqrt{3}$

$f''(x) = 12x^2-12 = 0 \Rightarrow x^2-1=0 \Rightarrow x=\pm 1$

a) critical points: $0, \sqrt{3}, -\sqrt{3}$

inflection points: $1, -1$

b)

x	$(-\infty,-1)$	-1	$(-1,1)$	1	$(1,+\infty)$
$f''(x)$	$f''(-2)=36$ $+$	0	$f''(0)=-12$ $-$	0	$f''(2)=36$ $+$
$f(x)$	$\cup$	I	$\cap$	I	$\cup$

f is concave down on $(-1, 1)$

So $f_{min} = -5$

c) $\lim\limits_{x\to+\infty} f(x) = +\infty \quad \lim\limits_{x\to-\infty} f(x) = +\infty$

$f(0) = 4 \quad f(\sqrt{3}) = 9-6\cdot3+4 = -5 \quad f(-\sqrt{3}) = 9-6\cdot3+4 = -5$

5. $\lim_{x\to 1} \frac{\ln x}{x^3-1} = \lim_{x\to 1} \frac{(\ln x)'}{(x^3-1)'} = \lim_{x\to 1} \frac{\frac{1}{x}}{3x^2} = \lim_{x\to 1} \frac{1}{3x^3} = \frac{1}{3}$

6. a) $\left.\begin{aligned} &\text{Area} = xy \\ &3x+4y=5 \Rightarrow y = \frac{5-3x}{4} \end{aligned}\right\} \Rightarrow$

$\text{Area} = A(x) = x \cdot \frac{5-3x}{4} = \frac{5x-3x^2}{4} = \frac{5}{4}x - \frac{3}{4}x^2$

b) $A'(x) = \frac{5}{4} - \frac{3}{2}x = 0 \Rightarrow x = \frac{5}{6} \Rightarrow y = \frac{5}{8}$

Since the x-intercept is $\frac{5}{3}$, then the range where x varies is $[0, \frac{5}{3}]$

$A_{max} = \max\{A(0), A(\frac{5}{3}), A(\frac{5}{6})\}$

$= \max\{0, \ 0, \ \frac{25}{48}\}$

$= \frac{25}{48}$

Therefore when $x = \frac{5}{6}$ $y = \frac{5}{8}$. the rectangle has the largest area $\frac{25}{48}$.