Quiz #9 Math 180 Name:

**Problem 1** (5pts). *Sketch the graph of the function* $f(x) = \sin x + \frac{1}{2}x$ *where* $x$ *belongs to* $[0, 2\pi]$.

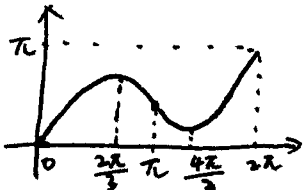
Step 1: $f'(x) = \cos x + \frac{1}{2} = 0 \Rightarrow \cos x = -\frac{1}{2} \Rightarrow x = \frac{2\pi}{3}, \frac{4\pi}{3}$

$f''(x) = (\cos x + \frac{1}{2})' = -\sin x = 0 \Rightarrow x = 0, \pi, 2\pi$

Step 2:

| $x$ | $0$ | $(0, \frac{2\pi}{3})$ | $\frac{2\pi}{3}$ | $(\frac{2\pi}{3}, \pi)$ | $\pi$ | $(\pi, \frac{4\pi}{3})$ | $\frac{4\pi}{3}$ | $(\frac{4\pi}{3}, 2\pi)$ | $2\pi$ |
|---|---|---|---|---|---|---|---|---|---|
| $f'(x)$ | | $f'(\frac{\pi}{2}) = \frac{1}{2}$ $+$ | $0$ | $f'(\frac{3\pi}{4}) = \frac{1-\sqrt{2}}{2}$ $-$ | | $f'(\frac{5\pi}{4}) = \frac{1-\sqrt{2}}{2}$ $-$ | $0$ | $f'(\frac{3\pi}{2}) = \frac{1}{2}$ $+$ | |
| $f''(x)$ | $0$ | $f''(\frac{\pi}{2}) = -1$ $-$ | | $f'(\frac{3\pi}{4}) = -\frac{\sqrt{2}}{2}$ $-$ | $0$ | $f'(\frac{5\pi}{4}) = \frac{\sqrt{2}}{2}$ $+$ | | $f'(\frac{3\pi}{2}) = 1$ $+$ | $0$ |
| $f(x)$ | $f(0) = 0$ | | Max | | $f(\pi) = \frac{\pi}{2}$ I | | Min | | $f(2\pi) = \pi$ |

Step 3:




**Problem 2** (5pts). *Sketch the graph of the function* $f(x) = \frac{1}{x} - \frac{1}{x-1}$

Step 1: $f'(x) = \left(\frac{1}{x} - \frac{1}{x-1}\right)' = \left(\frac{1}{x}\right)' - \left(\frac{1}{x-1}\right)' = -\frac{1}{x^2} - \left(-\frac{1}{(x-1)^2}\right)$

$= -\frac{1}{x^2} + \frac{1}{(x-1)^2}$

$f'(x) = 0 \Rightarrow -\frac{1}{x^2} + \frac{1}{(x-1)^2} = 0 \Rightarrow \frac{x^2 - (x-1)^2}{x^2(x-1)^2} = 0$

$\Rightarrow x^2 - (x-1)^2 = 0 \Rightarrow x = \frac{1}{2}$

$f''(x) = \left(-\frac{1}{x^2} + \frac{1}{(x-1)^2}\right)' = -\left[-\frac{1}{(x^2)^2} \cdot (x^2)'\right] + \left(-\frac{1}{[(x-1)^2]^2} \cdot [(x-1)^2]'\right)$

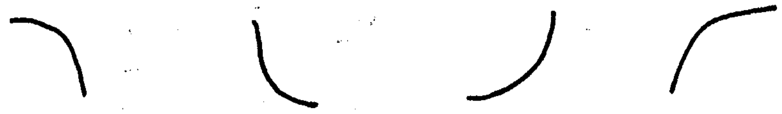
$= \frac{2}{x^3} - \frac{2}{(x-1)^3} = 2\,\frac{(x-1)^3 - x^3}{x^3(x-1)^3} = -2\,\frac{(x-1)^2 + x(x-1) + x^2}{x^3(x-1)^3}$

$f''(x) = 0 \Rightarrow 2(x-1)^3 - 2x^3 = 0 \Rightarrow (x-1)^3 - x^3 = 0$

$\Rightarrow [(x-1) - x][(x-1)^2 + x(x-1) + x^2] = 0 \Rightarrow$ No solution

because $(x-1)^2 + x(x-1) + x^2 > 0$ for any $x$ $(\neq 0 \,\&\, \neq 1)$

Step 2:

| $x$ | $(-\infty, 0)$ | $0$ | $(0, \frac{1}{2})$ | $\frac{1}{2}$ | $(\frac{1}{2}, 1)$ | $1$ | $(1, +\infty)$ |
|---|---|---|---|---|---|---|---|
| $f'(x)$ | $f'(-1) = -\frac{3}{4}$<br>$-$ | | $f'(\frac{1}{3}) = -\frac{27}{4}$<br>$-$ | $0$ | $f'(\frac{2}{3}) = \frac{27}{4}$<br>$+$ | | $f'(2) = \frac{3}{4}$<br>$+$ |
| $f''(x)$ | $f''(-1) = -\frac{+}{-\cdot-} = -$<br>$-$ | | $f''(\frac{1}{3}) = -\frac{+}{+\cdot-} = +$<br>$+$ | | $f''(\frac{2}{3}) = -\frac{+}{+\cdot-} = +$<br>$+$ | | $f''(2) = -\frac{+}{+\cdot+} = -$<br>$-$ |
| $f(x)$ |  | | | | | | |

Step 3: $\lim_{x\to\pm\infty} f(x) = \lim_{x\to\pm\infty} \frac{1}{x} - \lim_{x\to\pm\infty} \frac{1}{x-1} = 0 - 0 = 0$

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \left(\frac{1}{x} - \frac{1}{x-1}\right) = +\infty$

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \left(\frac{1}{x} - \frac{1}{x-1}\right) = -\infty$

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} \left(\frac{1}{x} - \frac{1}{x-1}\right) = -\infty$

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} \left(\frac{1}{x} - \frac{1}{x-1}\right) = +\infty$

Step 4: