Mathematics 180 - Hour Exam 1 - Spring 2009 - Solutions

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Solution for question 1.

(1) $\lim_{x\to 2} x^3 - 4 = 2^3 - 4 = 4$ because $f(x) = x^3 - 4$ is a composit function of basic functions, so is continuous.

(2) $\lim_{x\to 1} \frac{x^2-3x+2}{x^2-1} = \lim_{x\to 1} \frac{(x-1)(x-2)}{(x-1)(x+1)} = \lim_{x\to 1} \frac{x-2}{x+1} = \frac{1-2}{1+1} = -\frac{1}{2}$

(3) $\lim_{\theta\to 0} \frac{\sin 2\theta}{4\theta} = \frac{1}{2} \lim_{\theta\to 0} \frac{\sin 2\theta}{2\theta} = \frac{1}{2}$.

Solution for question 2.

(1) $f'(x) = (x^{11} - 2x^7 + 1)' = 11x^{10} - 14x^6$

(2) $f'(x) = (x^3 e^x)' = 3x^2 e^x + x^3 e^x$

(3) $f'(x) = (\frac{x-1}{x^2+2})' = \frac{(x^2+2) - 2x(x-1)}{(x^2+2)^2}$

Solution for question 3.

(1) Since $f(x)$ is a polynomial function which is continuous, $f(0) = 4 > 0$ and $f(1) = 1 - 6 + 4 = -1 < 0$, by intermidate value theorem, there exists a real number $c \in (0,1)$ such that $f(c) = 0$, i.e. f has a zero somewhere in $[0,1]$

(2) Consider the interval $[\frac{1}{2}, 1]$. Then $f(\frac{1}{2}) = \frac{1}{8} - 3 + 4 = 1\frac{1}{8} > 0$. As we've known, f is continous and $f(1) > 0$. Therefore f has a zero in $[\frac{1}{2}, 1]$ by IVT.

Solution for question 4.

By definition,

$$f'(3) = \lim_{h\to 0} \frac{f(3+h) - f(3)}{h} = \lim_{h\to 0} \frac{\frac{3}{3+h} - 1}{h} = \lim_{h\to 0} \frac{-h}{h(h+3)} = -\lim_{h\to 0} \frac{1}{h+3} = -\frac{1}{3}.$$

Solution for question 5.

(1) The average rate of change of f on the interval $[-1, 1]$ is

$$\frac{f(1) - f(-1)}{1 - (-1)} = \frac{(1^2 - 3\cdot 1) - ((-1)^2 - 3\cdot(-1))}{2} = \frac{-6}{2} = -3.$$

(2) The slope of the secant line through $(-1, f(-1))$ and $(1, f(2))$ is nothing but the average rate of change. Therefore the equation of the secant line is $y - f(1) = -3(x-1)$, equivalently $y = -3x + 1$.

(3) Since $f'(x) = 2x - 3$, then the slope of the tangent line at $x = -1$ is $f'(-1) = -5$. Using slope-point formula, we have the equation $y - f(-1) = -5(x - (-1))$, equivalently $y = -5x - 1$.

Solution for question 6. Here are some examples:

(1) $f(x) = |x|$ is continuous at $x = 0$, but the derivative at $x = 0$ does not exist. Because the left derivative at $x = 0$ is -1, while the right derivative at $x = 0$ is 1.

(2) $f(x) = x^{\frac{1}{2}}$ is continous at $x = 0$, but $f'(x) = \dfrac{1}{2\sqrt{x}}$ is undefined at $x = 0$.