Quiz #1 **Math 180 Spring 09** Name:

Note:Challenging Problems are for your own exercise.

Problem 1 ((4pts))**.** *Given a function $f(x) = x - x^2$.*

1. *Find the average rate of change over the following intervals:* $(0.99, 1)$, $(0.999, 1)$, $(0.9999, 1)$, $(1, 1.01)$, $(1, 1.001)$, $(1, 1.0001)$*;*
2. *Estimate the instantaneous ROC of $f(x)$ at $x = 1$;*
3. [1] *Find the tangent line of $f(x)$ at $x = 1$.*

Solution. *1.*

Interval	$(0.99, 1)$	$(0.999, 1)$	$(0.9999, 1)$	$(1, 1.01)$	$(1, 1.001)$	$(1, 1.0001)$
$\frac{f(x_1) - f(x_0)}{x_1 - x_0}$	-0.99	-0.999	-0.9999	-1.01	-1.001	-1.0001

2. The instantaneous ROC is -1.
3. Since the slope of tangent line equals the instantaneous ROC, i.e. -1, the defining equation of the tangent line is given by y-f(1)=-1(x-1), equivalently, y=-x+1.

Problem 2 ((6pts))**.** *Evaluate the limits using the Limit Laws.*

1. $\lim_{x\to 2} x^4 + 3x - 4x^2$*;*
2. $\lim_{h\to -1} \frac{h^{-3} - h^4}{h - 1}$*;*

Solution. *1.* $\lim_{x\to 2} x^4 + 3x - 4x^2 = (\lim_{x\to 2} x)^4 + 3\lim_{x\to 2} x - 4(\lim_{x\to 2} x)^2 = 16 + 6 - 16 = 6$.

2. $\lim_{h\to -1} \frac{h^{-3} - h^4}{h - 1} = \frac{(\lim_{h\to -1} h)^{-3} - (\lim_{h\to -1} h)^4}{(\lim_{h\to -1} h) - 1} = \frac{(-1)^{-3} - (-1)^4}{-1 - 1} = \frac{-1 - 1}{-1 - 1} = 1$

Problem 3 (Challenging Problem)**.** *Evaluate the limits. Assume that $\lim_{x\to a} f(x) = 1$ and $\lim_{x\to a} g(x) = 2$.*

[1] Challenging Problem. To find the tangent line is a kind of typical problem which you will see later. In fact, as you will see, the instantaneous ROC of a function at a point x is the same as the first derivative at x. They are both equal to the slope of the tangent line of the function at x.

1. $\lim\limits_{x\to a} \dfrac{xg(x)-2}{g(x)^2\sqrt{f(x)}-3}$;

2. $\lim\limits_{x\to 5a} f(\dfrac{x}{5})g(\dfrac{x}{5})+x$.

Solution. *1.* $\lim\limits_{x\to a} \dfrac{xg(x)-2}{g(x)^2\sqrt{f(x)}-3} = \dfrac{\lim\limits_{x\to a} x \lim\limits_{x\to a} g(x) - 2}{(\lim\limits_{x\to a} g(x))^2\sqrt{\lim\limits_{x\to a} f(x)}-3} = 2a-2.$

2. $\lim\limits_{x\to 5a} f(\dfrac{x}{5}) \lim\limits_{x\to 5a} g(\dfrac{x}{5}) + \lim\limits_{x\to 5a} x = \lim\limits_{t\to a} f(t) \lim\limits_{t\to a} g(t) + 5a = 2+5a,$ *where* $t=\frac{x}{5}$.