

Problem 1 (6 pts). Find the general antiderivatives of the following functions:

1. $f(x) = \sqrt{x}(x + 1)$;

2. $g(x) = \cos(2x - 1)$

Solution. 1. $\int \sqrt{x}(x + 1)dx = \int x^{\frac{3}{2}} + x^{\frac{1}{2}}dx = \frac{2}{5}x^{\frac{5}{2}} + \frac{2}{3}x^{\frac{3}{2}} + c$

2. $\int \cos(2x - 1) = \sin(2x - 1) + c$

Problem 2 (4pts). Solve the differential equation with initial condition.

$$\frac{dy}{dx} = e^{2x}, \quad y(0) = 2.$$

Solution. y is an antiderivative of e^{2x} with $y(0) = 2$. The general antiderivative of e^{2x} equals $\int e^{2x}dx = \frac{1}{2}e^{2x} + c$. Since y is one of the antiderivatives of e^{2x} , So $y = \frac{1}{2}e^{2x} + k$ for some constant k . Then that $y(0) = 2$ implies that $\frac{1}{2}e^{2 \cdot 0} + k = 2$. Then $k = 2 - \frac{1}{2}e^0 = 2 - \frac{1}{2} = \frac{3}{2}$.